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A retrial queueing-inventory system with service option on arrival and Multiple vacations

Jothivel Kathiresan[1], **Kathirvel Jeganathan**[2,*] **and Neelamegam Anbazhagan**[2]

[1]Department of Mathematics, Gnanamani College of Technology, Namakkal, India
[2]Ramanujan Institute for Advanced Study in Mathematics, University of Madras, Chennai, India
[3]Department of Mathematics, Alagappa University, Karaikudi, India

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Abstract. New models of a retrial queueing inventory system with finite buffer and two types of services (say, Type-1 and Type-2) are proposed. Here we discuss a type 1 service, in which a customer enters in the system and receives the service only. In the type-2 service, a customer gets the service and the level of inventory is reduced to one (demand an item). Some important system performance measures in the steady state are derived and long-run total expected cost rate of the proposed model is developed. The results of the numerical examples are shown .

Key words: Continuous review inventory system, Positive lead time, Retrial customers, Multiple vacations, Two types of service and Impatient customers.
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*Corresponding author K.Jeganathan: jegan.nathan85@yahoo.com
J.Kathiresan: kathiresanmat@gmail.com
N.Anbazhagan: n.anbazhagan.alu@gmail.com, anbazhagann@alagappauniversity.ac.in

Résumé (French) De nouveaux modèles de système de file d'attente relatifs aux inventaire avec mémoire tampon finie et deux types de services (par exemple, type 1 et type 2) sont proposés. Nous discutons un premier service de type 1 dans lequel un client entre dans le système et ne reçoit que le service. Dans le service de type 2, un client reçoit le service et ne peut demander qu'un article au plus. Certaines mesures importantes de performance du système en régime permanent sont donnees ainsi queque le taux de coût total prévu à long terme. Des résultats d'exemples numériques sont fournis.

1. Introduction

Queueing Inventory System (QIS) server vacation model is the most important in inventory management models. In inventory models, the concept of server vacation has been investigated by Daniel and Ramanarayanan (1987, 1988). Narayanan *et al.* (2008) analyzed Inventory System (IS) of stock out period when the server is on vacation period. Krishnamoorthy *et al.* (2010) developed a production IS with server vacation and service based on phase type distribution. Sivakumar (2011), Jeyaraman *et al.* (2012) and Padmavathi *et al.* (2015) have recently analysed the concept of server vacation models.

The inventory management system also plays an important role in many real life situations e.g. in manufacture, warehouse, supply chains and spare part allocation, etc. The mathematical study of QIS with service facility was initiated by Berman *et al.* (1993). Infinite capacity QIS with service facility was addressed by Berman and Kim (1999). Krishnamoorthy *et al.* (2011), Anbazhagan *et al.* (2014), Berman and Sapna (2000, 2001) have given more references of IS with service facility.

Recently, considerable attention has been given on inventory problems to study Retrial Queuing Inventory System (RQIS) by some of the researchers in different frame work. Artalejo *et al.* (2006) was the first worked on retrial demands in an IS. As a related work, Ushakumari (2006) considered RQIS with classical retrial policy. Yadavalli *et al.* (2018), Jeganathan *et al.* (2016, 2013) have studied RQIS in details. In many real life stuations (e.g. call centre and Barber shop setting), the customers impatience (amplified by large customers loads) leads naturally to large number of reneging. Ignoring the presence of reneging can lead to inappropriate sizing of the system. Some of the researchers have taken into account of the customer's impatient behavior in their works and poor staffing allocation. The present study points out some of the references Paul Manual *et al.* (2006), Yadavalli *et al.* (2015) and Rajkumar (2014).

Motivated by such practical situations, this paper considers RQIS, single server served at the two types of service, multiple vacations and impatient customers. Customers are arrived in a single server counter according to a Poisson process. The inventory is replenished according to (s, Q) policy and the replenishing times are assumed to be exponentially distributed. When the inventory level is zero, server goes to a vacation. If the server finds the empty stock at the end of a vacation, he immediately takes another vacation. Otherwise, the server starts his service on any arriving customers. When the waiting hall is full, any new arriving customer enters into the orbit of finite size and tries at a future time, which is exponentially distributed. The joint probability distribution of the number of customers in the orbit, the number of customers in the waiting hall, the server status and the inventory level is obtained for the steady state case. Various system performance measures in the steady state are derived and the long-run total expected cost rate is calculated.

The remainder of this paper is organized as follows. In section 2, the description of the mathematical model and notations are presented. Analysis of the model, the state equations governing the model are constructed in section 3. In section 4, various measures of system performance in steady state are derived. The total expected cost rate is calculated and cost function is carried out numerically in section 5. Last section gives the conclusion of the paper.

2. Model description

The behavior of the RQIS can be described as follows:

- Continuous time RQIS with maximum capacity of S units.
- As the (s, Q) replenishment policy, when on-hand inventory level drops to a prefixed level, say $s(> 0)$ an order for $Q(= S - s > s + 1)$ units is placed. The lead time is exponentially distributed with the rate β.
- Customers arrive in the system one by one in according to a Poisson stream with arrival rate $\lambda(> 0)$. The waiting room of the demands and the storage size are assumed to be N.
- Each arriving customer may choose either type 1 service with probability p_1 or type 2 service with probability p_2. The service time for both types are following exponential distribution with rate $p_i\mu(> 0)$, $i = 1, 2$.
- The completion of type-1 service, one customer receiving service only and type-2 service, one customer receiving service and reduces the inventory by one item.
- When the inventory level is zero, the server takes a vacation of random length. The vacation time also follows an exponential distribution with rate γ.
- On returning from vacation the server finds empty stock, he immediatly commences another vacation otherwise the server resumed to a regular service any arriving customers.
- Since an arriving customer will enter the orbit only if there are atmost N customers in the waiting hall, the maximum number of customers located in orbit is equal to M. Customers located in the orbit retry to enter the waiting hall with retrial rate θ. We assume constant retrial policy for the retrial customers.
- When the orbit is full, arriving primary customer is considered to be lost.
- If the waiting hall customer finds the server is on vacation, the customer becomes independently impatient and leaves the system with rate $\eta(> 0)$. which is exponentially distributed.

Notations

0	:	Zero matrix.
I	:	Identity matrix.
e	:	A column vector of $1'$s of appropriate dimension.
$[A]_{ij}$	:	Entry at (i, j)th position of a matrix A.
δ_{ij}	:	$\begin{cases} 1, & \text{if } i = j, \\ 0, & \text{otherwise.} \end{cases}$
$\bar{\delta}_{ij}$	:	$1 - \delta_{ij}$
$Z(t)$	:	$\begin{cases} 0, & \text{if server is on vacation at time } t. \\ 1, & \text{if server is not on vacation at time } t. \end{cases}$

3. Analysis

By the description of the model, we know that at time t, the state of the system considered in this paper can be described by the Markov process $\{(X(t), Y(t), Z(t), L(t)),\ t \geq 0\}$, where $X(t)$ denote the number of customers in the orbit at time t, $Y(t)$ is the number of customers in the waiting hall at time t, $Z(t)$ denotes the server status at time t and $L(t)$ represents the inventory level of the commodity at time t with state space given by E. Where

$E = \{(i, j, 0, l) : i = 0, 1, 2, \ldots, M, j = 0, 1, 2, \ldots, N, l = 0, Q\} \cup \{(i, j, 1, l) : i = 0, 1, 2, \ldots, M, j = 0, 1, 2, \ldots, N,\ l = 1, 2, \ldots, S\}$

To determine the infinitesimal generator

$$A = (a((i, j, k, l), (i^{'}, j^{'}, k^{'}, l^{'}))), \qquad (i, j, k, l), (i^{'}, j^{'}, k^{'}, l^{'}) \in E$$

of this process we use the following arguments :

$*$ An arrival of a primary customer makes a transition from $(i, j, 1, l)$ to $(i, j+1, 1, l)$ with intensity of transition λ, $i = 0, 1, 2, \ldots, M$, $j = 0, 1, 2, \ldots, N-1$, $l = 1, 2, \ldots, S$

$*$ An arrival of a primary customer makes a transition from $(i, j, 0, l)$ to $(i, j+1, 0, l)$ with intensity of transition λ, $i = 0, 1, 2, \ldots, M$, $j = 0, 1, 2, \ldots, N-1$, $l = 0, Q$

$*$ An arrival of a primary customer makes a transition from $(i, N, 1, l)$ to $(i+1, N, 1, l)$ with intensity of transition λ, $i = 0, 1, 2, \ldots, M-1$, $l = 1, 2, \ldots, S$

$*$ An arrival of a primary customer makes a transition from $(i, N, 0, l)$ to $(i+1, N, 0, l)$ with intensity of transition λ, $i = 0, 1, 2, \ldots, M-1$, $l = 0, Q$

$*$ The completion of type-1 service makes one customer leaves the system. Thus a transition takes place from $(i, j, 1, l)$ to $(i, j-1, 1, l)$ with intensity of transition $p_1\mu$, $i = 0, 1, 2, \ldots, M$, $j = 1, 2, \ldots, N$, $l = 1, 2, 3, \ldots, S$

* The completion of type-2 service makes one customer leaves the system and decreases the inventory level by one. Thus a transition takes place from $(i, j, 1, l)$ to $(i, j-1, 1, l-1)$ with intensity of transition $p_2\mu$, $i = 0, 1, 2, \ldots, M$, $j = 1, 2, \ldots, N$, $l = 2, 3, \ldots, S$

* The completion of type-2 service makes one customer leaves the system and decreases the inventory level by one. Thus a transition takes place from $(i, j, 1, 1)$ to $(i, j-1, 0, 0)$ with intensity of transition $p_2\mu$, $i = 0, 1, 2, \ldots, M$, $j = 1, 2, \ldots, N$

* A retrial requests of an orbiting customer makes a transition from (i, j, k, l) to $(i-1, j+1, k, l)$ with intensity of transition θ, $i = 1, 2, \ldots, M$, $j = 0$, $k = 0, 1$, $l = 0, 1, 2, \ldots, S$

* A reneging of a waiting hall customer makes a transition from $(i, j, 0, l)$ to $(i, j-1, 0, l)$ with intensity of transition η, $i = 0, 1, 2, \ldots, M$, $j = 1, 2, \ldots, N$, $l = 0, Q$

* When the vacation end, if the inventory level is positive, the server starts his service, otherwise the server takes another vacation. The intensity of transition from $(i, j, 0, l)$ to $(i, j, 1, l)$ is given by γ, $\quad i = 0, 1, \ldots, M$, $j = 0, 1, \ldots, N$, $l = Q$

* A transition from (i, j, k, l) to $(i, j, k, l+Q)$ for $i = 0, 1, \ldots, M$, $j = 0, 1, \ldots, N$, $k = 0$, $l = 0$ or for $i = 0, 1, \ldots, M$, $j = 0, 1, \ldots, N$, $k = 1$, $l = 1, 2, \ldots, s$ with intensity β when a replenishment occurs.

* We observe that no transition other than the above is possible.

* Finally, the value of $a((i, j, k, l), (i, j, k, l))$ is obtained by

$$a((i, j, k, l), (i, j, k, l)) = -\sum_{\substack{i'}}\sum_{j'}\sum_{k'}\sum_{\substack{l' \\ (i,j,k,l) \neq (i',j',k',l')}} a((i, j, k, l), (i', j', k', l')).$$

By ordering states lexicographically, the infinitesimal generator $A = (a((i, j, k, l), (i', j', k', l')))$, $\quad (i, j, k, l), (i', j', k', l') \in E$ can be conveniently expressed in a block partitioned matrix with entries

$$[A]_{ij} = \begin{cases} A_2, & j = i, \quad i = M \\ A_1, & j = i, \quad i = 1, 2, \dots, M-1 \\ A_0, & j = i, \quad i = 0 \\ B, & j = i+1, \quad i = 0, 1, 2, \dots, M-1 \\ C, & j = i-1, \quad i = 1, 2, \dots, M \\ \mathbf{0}, & \text{otherwise} \end{cases}$$

Where

$$[A_0]_{ij} = \begin{cases} D, & j = i, \quad i = 0 \\ E, & j = i, \quad i = 1, 2, \dots, N \\ F, & j = i+1, \quad i = 0, 1, 2, \dots, N-1 \\ G, & j = i-1, \quad i = 1, 2, \dots, N \\ \mathbf{0}, & \text{otherwise} \end{cases}$$

$$D = \begin{array}{c} \\ 1 \\ 0 \end{array} \!\!\begin{array}{c} \begin{array}{cc} 1 & \;\;\; 0 \end{array} \\ \begin{pmatrix} D_{11} & \mathbf{0} \\ D_{01} & D_{00} \end{pmatrix} \end{array}, \quad E = \begin{array}{c} \\ 1 \\ 0 \end{array} \!\!\begin{array}{c} \begin{array}{cc} 1 & \;\;\; 0 \end{array} \\ \begin{pmatrix} E_{11} & \mathbf{0} \\ E_{01} & E_{00} \end{pmatrix} \end{array}, \quad F = \begin{array}{c} \\ 1 \\ 0 \end{array} \!\!\begin{array}{c} \begin{array}{cc} 1 & \;\;\; 0 \end{array} \\ \begin{pmatrix} F_{11} & \mathbf{0} \\ \mathbf{0} & F_{00} \end{pmatrix} \end{array},$$

$$G = \begin{array}{c} \\ 1 \\ 0 \end{array} \!\!\begin{array}{c} \begin{array}{cc} 1 & \;\;\; 0 \end{array} \\ \begin{pmatrix} G_{11} & \mathbf{0} \\ \mathbf{0} & G_{00} \end{pmatrix} \end{array}$$

$$[D_{11}]_{kl} = \begin{cases} \beta, & l = k + Q, \quad k = s, s-1, \dots, 1 \\ -\lambda, & l = k, \quad l = S, S-1, \dots, s+1 \\ -(\lambda + \beta), & l = k, \quad k = s, s-1, \dots, 1 \\ 0, & \text{otherwise} \end{cases}$$

$$[D_{01}]_{kl} = \begin{cases} \gamma, & l = k, \quad k = Q \\ 0, & \text{otherwise} \end{cases}$$

$$[D_{00}]_{kl} = \begin{cases} \beta, & l = k + Q, \quad k = 0 \\ -(\lambda+\beta), & l = k, \quad l = 0 \\ -(\lambda+\gamma), & l = k, \quad k = Q \\ 0, & \text{otherwise} \end{cases}$$

$$[E_{11}]_{kl} = \begin{cases} \beta, & l = k + Q, \quad k = s, s-1, \dots, 1 \\ -(\lambda+p_1\mu+p_2\mu), & l = k, \quad l = S, S-1, \dots, s+1 \\ -(\lambda+\beta+p_1\mu+p_2\mu), & l = k, \quad k = s, s-1, \dots, 1 \\ 0, & \text{otherwise} \end{cases}$$

$$[E_{00}]_{kl} = \begin{cases} \beta, & l = k + Q, \quad k = 0 \\ -(\lambda+\beta+\eta), & l = k, \quad l = 0 \\ -(\lambda+\gamma+\eta), & l = k, \quad k = Q \\ 0, & \text{otherwise} \end{cases}$$

$$[F_{11}]_{kl} = \begin{cases} p_1\mu, & l = k, \quad k = S, S-1, \dots, 1 \\ p_2\mu, & l = k-1, \quad k = S, S-1, \dots, 1 \\ 0, & \text{otherwise} \end{cases}$$

$$[F_{00}]_{kl} = \begin{cases} \eta, & l = k, \quad k = 0, Q \\ 0, & \text{otherwise} \end{cases}$$

$$[G_{11}]_{kl} = \begin{cases} \lambda, & l = k, \quad k = S, S-1, \dots, 1 \\ 0, & \text{otherwise} \end{cases}$$

$$[G_{00}]_{kl} = \begin{cases} \lambda, & l = k, \quad k = 0, Q \\ 0, & \text{otherwise} \end{cases}$$

$$[A_1]_{ij} = \begin{cases} D_1, & j=i, \quad i=0 \\ E_1, & j=i, \quad i=1,2,\ldots,N \\ F_1, & j=i+1, \quad i=0,1,2,\ldots,N-1 \\ G_1, & j=i-1, \quad i=1,2,\ldots,N \\ \mathbf{0}, & \text{otherwise} \end{cases}$$

$$D_1 = \begin{array}{c c} & \begin{array}{cc} 1 & 0 \end{array} \\ \begin{array}{c} 1 \\ 0 \end{array} & \begin{pmatrix} D_{11}^{(1)} & \mathbf{0} \\ D_{01}^{(1)} & D_{00}^{(1)} \end{pmatrix} \end{array}, \quad E_1 = \begin{array}{c c} & \begin{array}{cc} 1 & 0 \end{array} \\ \begin{array}{c} 1 \\ 0 \end{array} & \begin{pmatrix} E_{11}^{(1)} & \mathbf{0} \\ E_{01}^{(1)} & E_{00}^{(1)} \end{pmatrix} \end{array}, \quad F_1 = \begin{array}{c c} & \begin{array}{cc} 1 & 0 \end{array} \\ \begin{array}{c} 1 \\ 0 \end{array} & \begin{pmatrix} F_{11}^{(1)} & \mathbf{0} \\ \mathbf{0} & F_{00}^{(1)} \end{pmatrix} \end{array},$$

$$G_1 = \begin{array}{c c} & \begin{array}{cc} 1 & 0 \end{array} \\ \begin{array}{c} 1 \\ 0 \end{array} & \begin{pmatrix} G_{11}^{(1)} & \mathbf{0} \\ \mathbf{0} & G_{00}^{(1)} \end{pmatrix} \end{array}$$

$$[D_{11}^{(1)}]_{kl} = \begin{cases} \beta, & l=k+Q, \quad k=s,s-1,\ldots,1 \\ -(\lambda+\theta), & l=k, \quad l=S,S-1,\ldots,s+1 \\ -(\lambda+\beta+\theta), & l=k, \quad k=s,s-1,\ldots,1 \\ 0, & \text{otherwise} \end{cases}$$

$$[D_{00}^{(1)}]_{kl} = \begin{cases} \beta, & l=k+Q, \quad k=0 \\ -(\lambda+\beta+\theta), & l=k, \quad l=0 \\ -(\lambda+\gamma+\theta), & l=k, \quad k=Q \\ 0, & \text{otherwise} \end{cases}$$

$$[A_2]_{ij} = \begin{cases} D_2, & j=i, \quad i=0 \\ E_2, & j=i, \quad i=1,2,\ldots,N-1 \\ E_3, & j=i, \quad i=N \\ F_2, & j=i+1, \quad i=0,1,2,\ldots,N-1 \\ G_2, & j=i-1, \quad i=1,2,\ldots,N \\ \mathbf{0}, & \text{otherwise} \end{cases}$$

$$D_2 = \begin{array}{c} \\ 1 \\ 0 \end{array} \begin{array}{c} \begin{array}{cc} 1 & \quad 0 \end{array} \\ \begin{pmatrix} D_{11}^{(2)} & \mathbf{0} \\ D_{01}^{(2)} & D_{00}^{(2)} \end{pmatrix} \end{array}, \quad E_2 = \begin{array}{c} \\ 1 \\ 0 \end{array} \begin{array}{c} \begin{array}{cc} 1 & \quad 0 \end{array} \\ \begin{pmatrix} E_{11}^{(2)} & \mathbf{0} \\ E_{01}^{(2)} & E_{00}^{(2)} \end{pmatrix} \end{array}, \quad E_3 = \begin{array}{c} \\ 1 \\ 0 \end{array} \begin{array}{c} \begin{array}{cc} 1 & \quad 0 \end{array} \\ \begin{pmatrix} E_{11}^{(3)} & \mathbf{0} \\ E_{01}^{(3)} & E_{00}^{(3)} \end{pmatrix} \end{array},$$

$$F_2 = \begin{array}{c} \\ 1 \\ 0 \end{array} \begin{array}{c} \begin{array}{cc} 1 & \quad 0 \end{array} \\ \begin{pmatrix} F_{11}^{(2)} & \mathbf{0} \\ \mathbf{0} & F_{00}^{(2)} \end{pmatrix} \end{array}, \quad G_2 = \begin{array}{c} \\ 1 \\ 0 \end{array} \begin{array}{c} \begin{array}{cc} 1 & \quad 0 \end{array} \\ \begin{pmatrix} G_{11}^{(2)} & \mathbf{0} \\ \mathbf{0} & G_{00}^{(2)} \end{pmatrix} \end{array}$$

$$[E_{11}^{(3)}]_{kl} = \begin{cases} \beta, & l = k + Q, \quad k = s, s-1, \ldots, 1 \\ -(p_1\mu + p_2\mu), & l = k, \quad l = S, S-1, \ldots, s+1 \\ -(\beta + p_1\mu + p_2\mu), & l = k, \quad k = s, s-1, \ldots, 1 \\ 0, & \text{otherwise} \end{cases}$$

$$[E_{00}^{(3)}]_{kl} = \begin{cases} \beta, & l = k + Q, \quad k = 0 \\ -(\beta + \eta), & l = k, \quad l = 0 \\ -(\gamma + \eta), & l = k, \quad k = Q \\ 0, & \text{otherwise} \end{cases}$$

$D_{01} = D_{01}^{(1)} = D_{01}^{(2)}$, $E_{01} = E_{01}^{(1)} = E_{01}^{(2)} = E_{01}^{(3)}$, $E_{11} = E_{11}^{(1)} = E_{11}^{(2)}$, $E_{00} = E_{00}^{(1)} = E_{00}^{(2)}$.

$$[B]_{ij} = \begin{cases} B_1, & j = i, \quad i = N \\ \mathbf{0}, & \text{otherwise} \end{cases}$$

$$B_1 = \begin{array}{c} \\ 1 \\ 0 \end{array} \begin{array}{c} \begin{array}{cc} 1 & \quad 0 \end{array} \\ \begin{pmatrix} B_{11} & \mathbf{0} \\ \mathbf{0} & B_{00} \end{pmatrix} \end{array},$$

$$[B_{11}]_{kl} = \begin{cases} \lambda, & l = k, \quad k = S, S-1, S-2, \ldots, 1 \\ 0, & \text{otherwise} \end{cases}$$

$$[B_{00}]_{kl} = \begin{cases} \lambda, & l = k, \quad k = 0, Q \\ 0, & \text{otherwise} \end{cases}$$

$$[C]_{ij} = \begin{cases} C_1, & j = i, \qquad i = N \\ \\ \mathbf{0}, & \text{otherwise} \end{cases}$$

$$C_1 = \begin{array}{c} \\ 1 \\ 0 \end{array} \begin{array}{c} \begin{array}{cc} 1 & \quad 0 \end{array} \\ \begin{pmatrix} C_{11} & \mathbf{0} \\ \mathbf{0} & C_{00} \end{pmatrix} \end{array},$$

$$[C_{11}]_{kl} = \begin{cases} \theta, & l = k, \qquad k = S, S-1, S-2, \ldots, 1 \\ \\ 0, & \text{otherwise} \end{cases}$$

$$[C_{00}]_{kl} = \begin{cases} \theta, & l = k, \qquad k = 0, Q \\ \\ 0, & \text{otherwise} \end{cases}$$

3.1. Steady State Analysis

It can be seen from the structure of A that the homogeneous Markov process $\{(X(t), Y(t), Z(t), L(t)),\ t \geq 0\}$ on the finite state space E is irreducible. Hence the limiting distribution

$\pi^{(i_1,i_2,i_3,i_4)} = \lim_{t\to\infty} pr\{X(t) = i_1, Y(t) = i_2, Z(t) = i_3, L(t) = i_4 | X(0), Y(0), Z(0), L(0)\}$,
exists. Let

$$\mathbf{\Pi} = (\mathbf{\Pi^{(0)}}, \mathbf{\Pi^{(1)}}, \mathbf{\Pi^{(2)}}, \ldots, \mathbf{\Pi^{(M)}})$$

we partition the vector, $\mathbf{\Pi^{(i_1)}}$ into as follows:

$$\mathbf{\Pi^{(i_1)}} = (\mathbf{\Pi^{(i_1,0)}}, \mathbf{\Pi^{(i_1,1)}}, \mathbf{\Pi^{(i_1,2)}}, \ldots, \mathbf{\Pi^{(i_1,N)}}), \quad i_1 = 0, 1, 2, \ldots, M$$

which is partitioned as follows:

$$\mathbf{\Pi^{(i_1,i_2)}} = (\pi^{(i_1,i_2,0,0)}, \pi^{(i_1,i_2,0,Q)}, \pi^{(i_1,i_2,1,1)}, \pi^{(i_1,i_2,1,2)}, \ldots, \pi^{(i_1,i_2,1,S)})$$

for $i_1 = 0, 1, 2, \ldots, M, \quad i_2 = 0, 1, 2, \ldots, N$
Then the limiting probability, $\mathbf{\Pi}$ satisfies

$$\mathbf{\Pi} A = 0, \qquad \mathbf{\Pi} e = 1 \tag{1}$$

From the structure of A, it is a finite QBD matrix, therefore its steady state vector $\mathbf{\Pi}$ can be computed by using the following algorithm described by Gaver *et al.* (1984).

Algorithm :

1. Determine recursively the matrices

$$F_0 = A_0$$

$$F_i = A_1 + B(-F_{i-1}^{-1})C, \qquad i = 1, 2, \ldots, M-1,$$

$$F_M = A_2 + B(-F_{M-1}^{-1})C.$$

2. Compute recursively the vectors $\mathbf{\Pi}^{(i)}$ using

$$\mathbf{\Pi}^{(i)} = \mathbf{\Pi}^{(i+1)} B(-F_i^{-1}), \qquad i = 0, 1, 2, \ldots, M-1$$

3. Solve the system of equations

$$\mathbf{\Pi}^{(M)} F_M = 0 \text{ and } \sum_{i=0}^{M} \mathbf{\Pi}^{(i)} e = 1.$$

From the system of equations $\mathbf{\Pi}^{(M)} F_M = 0$, vector $\mathbf{\Pi}^{(M)}$ could be determined uniquely, upto a multiplicative constant. This constant is decided by
$\mathbf{\Pi}^{(i)} = \mathbf{\Pi}^{(i+1)} B(-F_i^{-1}), \quad i = 0, 1, 2, \ldots, M-1$ and $\sum_{i=0}^{M} \mathbf{\Pi}^{(i)} e = 1.$

4. System Performance Measures

In this section, we establish some performance indices, which are of great interest in order to determine the performance and efficiency of the system and are given below:

4.1. Expected inventory level

Let ρ_I denote the mean inventory level in the steady state. Then

$$\rho_I = \sum_{i_1=0}^{M} \sum_{i_2=0}^{N} \left[\sum_{i_4=1}^{S} i_4 \pi^{(i_1,i_2,1,i_4)} + Q\pi^{(i_1,i_2,0,Q)} \right]$$

4.2. Expected reorder rate

Let ρ_R denote the expected reorder rate in the steady state. Then

$$\rho_R = \sum_{i_1=0}^{M} \sum_{i_2=1}^{N} p_2 \mu \left[\pi^{(i_1,i_2,1,s+1)} \right]$$

4.3. *Expected number of demands in the waiting hall*

Let ρ_W denote the expected number of demands in the waiting hall in the steady state. Then

$$\rho_W = \sum_{i_1=0}^{M} \sum_{i_2=1}^{N} \sum_{i_4=1}^{S} i_2 \left[\pi^{(i_1,i_2,1,i_4)} + \pi^{(i_1,i_2,0,0)} + \pi^{(i_1,i_2,0,Q)} \right]$$

4.4. *Expected number of demands in the orbit*

Let ρ_O denote the expected number of demands in the orbit in the steady state. Then

$$\rho_O = \sum_{i_1=1}^{M} \sum_{i_2=0}^{N} \sum_{i_4=1}^{S} i_1 \left[\pi^{(i_1,i_2,1,i_4)} + \pi^{(i_1,i_2,0,0)} + \pi^{(i_1,i_2,0,Q)} \right]$$

4.5. *Overall rate of retrials*

Let ρ_{OR} denote the overall rate of retrials in the steady state. Then

$$\rho_{OR} = \sum_{i_1=1}^{M} \sum_{i_2=0}^{N} \sum_{i_4=1}^{S} \theta \left[\pi^{(i_1,i_2,1,i_4)} + \pi^{(i_1,i_2,0,0)} + \pi^{(i_1,i_2,0,Q)} \right]$$

4.6. *The successful retrial rate*

Let ρ_{SR} denote the successful retrial rate in the steady state. Then

$$\rho_{SR} = \sum_{i_1=1}^{M} \sum_{i_4=1}^{S} \theta \left[\pi^{(i_1,0,1,i_4)} + \pi^{(i_1,0,0,0)} + \pi^{(i_1,0,0,Q)} \right]$$

4.7. *The fraction of successful rate of retrial*

Let ρ_{FR} denote the successful retrial rate in the steady state. Then

$$\rho_{FR} = \frac{\rho_{SR}}{\rho_{OR}}$$

4.8. *Fraction of time server is on vacation*

Let ρ_{FV} denote the server is on vacation in the steady state. Then

$$\rho_{FV} = \sum_{i_1=0}^{M} \sum_{i_2=0}^{N} \left[\pi^{(i_1,i_2,0,0)} + \pi^{(i_1,i_2,0,Q)} \right]$$

4.9. Expected blocking rate

Let ρ_B denote the expected blocking rate in the steady state. Then

$$\rho_B = \sum_{i_1=1}^{M-1} \sum_{i_4=0}^{S} \lambda \left[\pi^{(i_1,N,1,i_4)} + \pi^{(i_1,N,0,0)} + \pi^{(i_1,N,0,Q)} \right]$$

4.10. Expected reneging rate

Let ρ_{Rg} denote the expected reneging rate in the steady state. Then

$$\rho_{Rg} = \sum_{i_1=0}^{M} \sum_{i_2=1}^{N} \eta \left[\pi^{(i_1,i_2,0,0)} + \pi^{(i_1,i_2,0,Q)} \right]$$

5. Cost Analysis

The expected total cost per unit time (expected total cost rate) in the steady state for this model is defined to be

c_h : the inventory holding cost per unit item per unit time

c_s : the inventory setup cost per unit item per unit time

c_b : cost per blocking customer

c_w : Waiting cost of a customer in the waiting hall per unit time.

c_o : Waiting cost of a customer in the orbit per unit time.

c_r : reneging cost per customer per unit time.

The long run total expected cost rate is given by

$$TC(S, s, N, M) = c_h \rho_I + c_s \rho_R + c_b \rho_B + c_w \rho_W + c_o \rho_O + c_r \rho_{Rg}$$

Substituting ρ's into the above equation, we obtain

$$TC(S,s,N,M) = c_h \sum_{i_1=0}^{M}\sum_{i_2=0}^{N}\left[\sum_{i_4=1}^{S} i_4 \pi^{(i_1,i_2,1,i_4)} + Q\pi^{(i_1,i_2,0,Q)}\right] + c_s$$
$$\sum_{i_1=0}^{M}\sum_{i_2=1}^{N} p_2\mu\left[\pi^{(i_1,i_2,1,s+1)}\right] + c_b \sum_{i_1=1}^{M-1}\sum_{i_4=0}^{S}\lambda\left[\pi^{(i_1,N,1,i_4)} + \pi^{(i_1,N,0,0)} + \pi^{(i_1,N,0,Q)}\right]$$
$$+c_w \sum_{i_1=0}^{M}\sum_{i_2=1}^{N}\sum_{i_4=1}^{S} i_2\left[\pi^{(i_1,i_2,1,i_4)} + \pi^{(i_1,i_2,0,0)} + \pi^{(i_1,i_2,0,Q)}\right] + c_o \sum_{i_1=1}^{M}\sum_{i_2=0}^{N}\sum_{i_4=1}^{S} i_1$$
$$\left[\pi^{(i_1,i_2,1,i_4)} + \pi^{(i_1,i_2,0,0)} + \pi^{(i_1,i_2,0,Q)}\right] + c_r \sum_{i_1=0}^{M}\sum_{i_2=1}^{N}\eta\left[\pi^{(i_1,i_2,0,0)} + \pi^{(i_1,i_2,0,Q)}\right]$$

Due to the complex form of the limiting distribution, it is difficult to discuss the qualitative behavior of the cost function $TC(S,s,N,M)$ analytically. Hence, a detailed computational study of the total expected cost rate function is carried out in the next section.

5.1. Numerical Examples

In this section, we investigate some numerical examples that reveal the possible convexity of the total expected cost rate. We explore the behavior of the cost function by considering it as function of any two variables by fixing the others are constants.

Table 1 and 2 give the total expected cost rate as a function of $TC(S,s,9,4)$ and $TC(S,4,N,4)$. All costs and other parameters are assigned fixed values which are indicated in each table. The value that is shown bold is the least among the values in that column and the value that is shown underlined is the least in that row. It may be observed that, these values in each Table exhibit a (possibly) local minimum of the function of the two variables.
Example 1: In this example, we investigate the effects of some parameters on the performance measures of the system and show an optimum value in terms of the cost function TC. From table 3 - 6, the variations of ρ_I, ρ_R, ρ_W, ρ_O, ρ_B, ρ_{Rg} and TC are presented for various parameters $\lambda = 4$, $\theta = 1.2$, $\beta = 1.5$, $\mu = 1.6$, $\eta = 0.02$, $\gamma = 0.58$, $p_1 = 0.4$ and $p_2 = 0.6$. We observe the following from Table 3-6.

1. The system performance measures ρ_I, ρ_R, ρ_W, ρ_O, TC increase and ρ_B decreases, when arrival rate λ increases.
2. The system performance measures ρ_I, ρ_R, ρ_B, ρ_{Rg}, TC increase and ρ_W, ρ_O decrease, when service rate μ increases.
3. The system performance measures ρ_I, ρ_W, ρ_B, TC increase and ρ_O decreases, when retrial rate θ increases.
4. The system performance measures ρ_{Rg}, TC increase and ρ_W decreases, when renaging rate η increases.

Table 1. Total expected cost rate as a function of S and s

$\lambda = 4,\ \theta = 1.2,\ p_1 = 0.2,\ p_2 = 0.8,\ \beta = 0.5,\ \gamma = 0.58,\ \mu = 1.6,\ \eta = 0.02,$
$c_h = 0.03,\ c_s = 13,\ c_w = 0.5,\ c_b = 0.02,\ c_o = 0.2,\ c_r = 1,\ N = 9,\ M = 4$

S \ s	8	9	10	11	12
36	6.324196	<u>6.314760</u>	6.320013	6.336561	6.362024
37	**6.323828**	<u>6.311477</u>	6.314026	6.327946	6.350744
38	6.324784	<u>6.309677</u>	6.309702	6.321201	6.341576
39	6.326938	**6.309213**	<u>6.306871</u>	6.316128	6.334285
40	6.330179	6.309957	<u>6.305384</u>	6.312553	6.328671
41	6.334410	6.311795	<u>**6.305111**</u>	6.310329	6.324561
42	6.339543	6.314630	<u>6.305940</u>	**6.309324**	6.321806
43	6.345502	6.318375	<u>6.307770</u>	6.309425	6.320274
44	6.352219	6.322951	<u>6.310514</u>	6.310532	**6.319851**
45	6.359633	6.328289	6.314094	<u>6.312555</u>	6.320436

Table 2. Total expected cost rate as a function of S and N

$\lambda = 4,\ \theta = 1.2,\ p_1 = 0.4,\ p_2 = 0.6,\ \beta = 1.5,\ \gamma = 0.58,\ \mu = 1.6,\ \eta = 0.02,\ c_h = 0.3,\ c_s = 13,$
$c_w = 0.05\ ,c_b = 3.02,\ c_o = 0.2,\ c_r = 1,\ s = 4,\ M = 4$

S \ N	9	10	11	12	13
10	5.0703484	<u>5.0703455</u>	5.0703469	5.0703485	5.0703494
11	4.9410391	4.9410297	<u>4.9410287</u>	4.9410292	4.9410299
12	4.8839526	4.8839402	<u>4.8839380</u>	4.8839381	4.8839386
13	**4.8751560**	**4.8751429**	<u>**4.8751405**</u>	**4.8751406**	**4.8751410**
14	4.9002389	4.9002269	<u>4.9002250</u>	4.9002253	4.9002258
15	4.9500059	4.9499963	<u>4.9499953</u>	4.9499961	4.9499968
16	5.0183132	<u>5.0183069</u>	5.0183074	5.0183087	5.0183096

5. The system performance measures ρ_R increases and ρ_I, ρ_W, ρ_O, ρ_{Rg}, TC decrease, when reorder rate β increases.

Example 2: In this example, we study the impact of c_h, c_s, c_w, c_b, c_o and c_r on the optimal values (s^*, S^*) and the corresponding total expected cost rate TC^* . By fixing the parameter values as $\lambda = 4$, $\theta = 1.2$, $p_1 = 0.4$, $p_2 = 0.6$, $\beta = 1.5$, $\gamma = 0.58$, $\mu = 1.6$, $\eta = 0.02$. We observe the following from table 8 to 13.

1. The Total expected cost rate increases when c_h, c_s, c_w, c_b, c_o and c_r increase.
2. If c_s increases, then S^* monotonically increases. If c_h increases, then S^* monotonically decreases.

Table 3. Effects of λ and μ on some performance measures and total cost.

λ	μ	ρ_I	ρ_R	ρ_W	ρ_O	ρ_B	ρ_{Rg}	TC
5	1.5	26.162812	0.038539	8.573648	3.999967	0.000013	0.000088	6.372794
	1.6	26.727563	0.041056	8.532599	3.999942	0.000022	0.000113	6.401958
	1.7	27.401129	0.043559	8.489319	3.999901	0.000037	0.000142	6.433081
6	1.5	26.163105	0.038539	8.668278	3.999995	0.000003	0.000088	6.420126
	1.6	26.728066	0.041057	8.638636	3.999990	0.000005	0.000113	6.455009
	1.7	27.402048	0.043560	8.607775	3.999983	0.000008	0.000142	6.492365
7	1.5	26.163163	0.038539	8.728554	3.999999	0.000001	0.000088	6.450267
	1.6	26.728178	0.041057	8.705496	3.999998	0.000001	0.000113	6.488445
	1.7	27.402285	0.043560	8.681688	3.999996	0.000002	0.000142	6.529333

$\theta = 1.2$, $p_1 = 0.2$, $p_2 = 0.8$, $\beta = 0.5$, $\gamma = 0.58$, $\eta = 0.02$, $c_h = 0.03$, $c_s = 13$, $c_w = 0.5$, $c_b = 0.02$, $c_o = 0.2$, $c_r = 1$, $N = 9$, $M = 4$

Table 4. Effects of λ and β on some performance measures and total cost.

λ	β	ρ_I	ρ_R	ρ_W	ρ_O	ρ_B	ρ_{Rg}	TC
5	0.6	25.771375	0.041166	8.531201	3.999941	0.000023	0.000060	6.373951
	0.7	25.305808	0.041222	8.530490	3.999941	0.000023	0.000033	6.360316
	0.8	25.092866	0.041250	8.530109	3.999940	0.000023	0.000019	6.354099
6	0.6	25.771715	0.041167	8.637593	3.999990	0.000005	0.000060	6.427176
	0.7	25.306126	0.041222	8.637066	3.999990	0.000005	0.000033	6.413631
	0.8	25.093119	0.041251	8.636784	3.999990	0.000005	0.000019	6.407462
7	0.6	25.771860	0.041167	8.704663	3.999998	0.000001	0.000060	6.460717
	0.7	25.306201	0.041222	8.704243	3.999998	0.000001	0.000033	6.447225
	0.8	25.093179	0.041251	8.704020	3.999998	0.000001	0.000019	6.441055

$\mu = 1.6$, $\theta = 1.2$, $p_1 = 0.2$, $p_2 = 0.8$, $\gamma = 0.58$, $\eta = 0.02$, $c_h = 0.03$, $c_s = 13$, $c_w = 0.5$, $c_b = 0.02$, $c_o = 0.2$, $c_r = 1$, $N = 9$, $M = 4$

Table 5. Effects of λ and η on some performance measures and total cost.

λ	η	ρ_I	ρ_R	ρ_W	ρ_O	ρ_B	ρ_{Rg}	TC
5	0.02	26.727563	0.041056	8.532599	3.999942	0.000022	0.000113	6.401958
	0.03	26.727563	0.041056	8.532585	3.999942	0.000022	0.000170	6.402008
	0.04	26.727563	0.041056	8.532572	3.999942	0.000022	0.000226	6.402058
6	0.02	26.728066	0.041057	8.638636	3.999990	0.000005	0.000113	6.455009
	0.03	26.728066	0.041057	8.638625	3.999990	0.000005	0.000170	6.455060
	0.04	26.728066	0.041057	8.638615	3.999990	0.000005	0.000226	6.455111
7	0.02	26.728178	0.041057	8.705496	3.999998	0.000001	0.000113	6.488445
	0.03	26.728178	0.041057	8.705487	3.999998	0.000001	0.000170	6.488497
	0.04	26.728178	0.041057	8.705478	3.999998	0.000001	0.000226	6.488549

$\mu = 1.6$, $\theta = 1.2$, $p_1 = 0.2$, $p_2 = 0.8$, $\gamma = 0.58$, $\beta = 0.5$, $c_h = 0.03$, $c_s = 13$, $c_w = 0.5$, $c_b = 0.02$, $c_o = 0.2$, $c_r = 1$, $N = 9$, $M = 4$

Table 6. Effects of θ and μ on some performance measures and total cost.

θ	μ	ρ_I	ρ_R	ρ_W	ρ_O	ρ_B	ρ_{Rg}	TC
1.3	1.5	26.161342	0.038537	8.403972	3.999694	0.000087	0.000088	6.287830
	1.6	26.724719	0.041052	8.339372	3.999457	0.000149	0.000113	6.305117
	1.7	27.359973	0.043552	8.269892	3.999073	0.000245	0.000141	6.322967
1.4	1.5	26.161372	0.038537	8.403982	3.999676	0.000092	0.000088	6.287834
	1.6	26.724776	0.041053	8.339390	3.999426	0.000158	0.000113	6.305123
	1.7	27.396076	0.043553	8.269921	3.999020	0.000259	0.000141	6.322976
1.5	1.5	26.161400	0.038537	8.403993	3.999659	0.000097	0.000088	6.287837
	1.6	26.724830	0.041053	8.339407	3.999396	0.000166	0.000113	6.305128
	1.7	27.396175	0.043553	8.269949	3.998969	0.000273	0.000141	6.322985
$\lambda = 4, p_1 = 0.2, p_2 = 0.8, \gamma = 0.58, \beta = 0.5, \eta = 0.02, c_h = 0.03, c_s = 13, c_w = 0.5, c_b = 0.02, c_o = 0.2, c_r = 1, N = 9, M = 4$								

Table 7. Effect of c_h and c_s on optimal values

c_s / c_h	12	13	14	15	16
0.029	40 10 6.236830	41 10 6.278386	42 10 6.318408	44 10 6.356938	45 10 6.394190
0.030	40 10 6.262971	41 10 6.305111	42 10 6.345716	43 10 6.384925	44 10 6.422860
0.031	39 10 6.288563	41 10 6.331525	41 10 6.372888	42 10 6.412800	44 10 6.451334
0.032	39 10 6.314121	40 10 6.357667	41 10 6.399613	42 10 6.440108	43 10 6.479284
0.033	38 10 6.339204	39 10 6.383546	40 10 6.426221	41 10 6.467390	43 10 6.507174

Table 8. Effect of c_s and c_w on optimal values

c_w / c_s	0.4	0.5	0.6	0.7	0.8
12	40 10 5.389020	40 10 6.262971	40 10 7.096922	40 10 7.930873	40 10 8.764824
13	41 10 5.471176	41 10 6.305111	41 10 7.139046	41 10 7.972982	41 10 8.806917
14	42 10 5.511795	42 10 6.345716	42 10 7.179637	42 10 8.013558	42 10 8.847479
15	43 10 5.551018	43 10 6.384925	43 10 7.218832	43 10 8.052739	43 10 8.886647
16	44 10 5.588966	44 10 6.422860	44 10 7.256754	44 10 8.090649	44 10 8.924543

Table 9. Effect of c_w and c_b on optimal values

c_w \ c_b	0.02	0.03	0.04	0.05	0.06
0.4	41 10 5.471176	41 10 5.471177	41 10 5.471178	41 10 5.471180	41 10 5.471181
0.5	41 10 6.305111	41 10 6.305112	41 10 6.305114	41 10 6.305115	41 10 6.305117
0.6	41 10 7.139046	41 10 7.139048	41 10 7.139049	41 10 7.139051	41 10 7.139052
0.7	41 10 7.972982	41 10 7.972985	41 10 7.972985	41 10 7.972986	41 10 7.972987
0.8	41 10 8.806917	41 10 8.806919	41 10 8.806920	41 10 8.806921	41 10 8.806923

Table 10. Effect of c_w and c_o on optimal values

c_w \ c_o	0.2	0.3	0.4	0.5	0.6
0.4	41 10 5.471176	41 10 5.871125	41 10 6.271074	41 10 6.671023	41 10 7.070972
0.5	41 10 6.305111	41 10 6.705660	41 10 7.105009	41 10 7.504958	41 10 7.904907
0.6	41 10 7.139046	41 10 7.538995	41 10 7.938944	41 10 8.338893	41 10 8.738842
0.7	41 10 7.972982	41 10 8.372931	41 10 8.772880	41 10 9.172829	41 10 9.572778
0.8	41 10 8.806917	41 10 9.206866	41 10 9.606815	41 10 10.006764	41 10 10.406713

Table 11. Effect of c_w and c_r on optimal values

c_w \ c_r	0.9	1.0	1.1	1.2	1.3
0.4	41 10 5.471164	41 10 5.471176	41 10 5.471187	41 10 5.471198	41 10 5.471210
0.5	41 10 6.305100	41 10 6.305111	41 10 6.305122	41 10 6.305134	41 10 6.305145
0.6	41 10 7.139035	41 10 7.139046	41 10 7.139058	41 10 7.139069	41 10 7.139080
0.7	41 10 7.972971	41 10 7.972982	41 10 7.972993	41 10 7.973004	41 10 7.973016
0.8	41 10 8.806906	41 10 8.806917	41 10 8.806929	41 10 8.806940	41 10 8.806951

6. Conclusion and Perspectives

We have proposed a new model of stochastic inventory system with finite buffer. We discussed in this paper two different types of services. In type-1 service we assumed, a customer enters into the system and receiving the service only (not demand an item). In type-2 service we assumed, a customer getting the service and the level of inventory is reduced by one (demand an item). The model is analyzed within the framework of Markov processes. Joint probability distribution of the number of customers in the orbit, the number of customers in the waiting hall, the server status and the inventory level is obtained for the steady state case. Various system performance measures in the steady state are derived and the long-run total expected cost rate is calculated. By assuming a appropriate cost structure on the queueing-inventory system, we have presented extensive numerical illustrations to show the effect of change of values for constants on the total expected cost rate. The authors are working in the direction of MAP (Markovian arrival process) arrival for the customers and service times that follow PH-distributions.

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References

Anbazhagan N., Vigneshwaran B., and Jeganathan K., 2014. Stochastic inventory system with two types of services. *International journal of advanced applied mathematics and mechanics*, 2(1), 120-127.

Artalejo, J. R., Krishnamoorthy, A. and Lopez-Herrero, M. J., 2006. Numerical analysis of (s, S) inventory systems with repeated attempts. *Annals of Operations Research*, 141, 67-83.

Berman, O., Kaplan, E. H. and Shimshak, D. G., 1993. Deterministic approximations for inventory management at service facilities. *IIE Transactions*, 25, 98-104.

Berman, O. and Kim, E., 1999. Stochastic models for inventory management of service facilities. *Stochastic Models*, 15, 695-718.

Berman, O. and Sapna, K.P., 2000. Inventory management at service facility for systems with arbitrarily distributed service times. *Communications in Statistics. Stochastic Models*, 16(3), 343-360.

Berman, O. and Sapna, K.P., 2001. Optimal control of service for facilities holding inventory. *Computers and Operations Research*, 28(5), 429-441.

Daniel J. K. and Ramanarayanan R., 1987. An inventory system with two servers and rest periods. *Cahiers du C.E.R.O, Universite Libre De Bruxelles*, 29, 95-100.

Daniel J. K. and Ramanarayanan R., 1988. An inventory system with rest periods to the server. *Naval Research Logistics*, 35, 119-123.

Gaver D.P., P. A. Jacobs and G. Latouche, 1984. Finite birth-and-death models in randomly changing environments. *Advances in Applied Probability*, 16, 715 - 731.

Jeganathan, K. Anbazhagan, N. and Kathiresan, J., 2013. A retrial inventory system with non-preemptive priority Service. *International Journal of Information and Management Sciences*, 24, 57-77.

Jeganathan, K. Kathiresan, J. and Anbazhagan, N., 2016. A retrial inventory system with priority customers and second optional service. *OPSEARCH*, 53(4), 808-834.

Jeyaraman, R., Sivakumar, B. and Arivarignan, G., 2012. A perishable inventory system with postponed demands and multiple server vacations. *Modelling and Simulation Engineering*, Article ID 620960, 17 pages, DOI: 10.1155/2012/620960.

Krishnamoorthy, A. Lakshmy, B. and Manikandan, R., 2011. A survey on inventory models with positive service time. *OPSEARCH*, 48(2), 153 - 169.

Krishnamoorthy, A., viswanath, and Narayanan, C., 2010. Production inventory with service time and vacation to the server. *IMA Journal of Management Mathematics*, 1-13, DOI: 10.1093/imaman/dpp025.

Narayanan V. C., Deepak T. G., Krishnamoorthy A. and Krishnakumar B., 2008. On an (s, S) inventory policy with service time, vacation to server and correlated lead time. *Qualitative Technology and Quantitative Management*, 5(2), 129-143.

Padmavathi, I., Sivakumar, B. and Arivarignan, G., 2015. A retrial inventory system with single and modified multiple vacation for server, *Annals of Operation Research*, 233(1), pp.335-364.

Paul Manual, Sivakumar, B., and Arivarignan, G., 2006. Service facility inventory system with impatient customers. *International Journal of Mathematics, Game Theory and Algebra*, 15(4), 355-367.

Rajkumar, M., 2014. An (s, S) retrial inventory system with impatient and negative customers. *International Journal of Mathematics in Operational Research*, 6(1), 106-122.

Sivakumar, B., 2011. An Inventory System with Retrial Demands and Multiple Server Vacation. *Quality Technology and Quantitative Management*, 8(2), 125-146.

Ushakumari, P.V., 2006. On (s,S) inventory system with random lead time and repeated demands. *Journal of Applied Mathematics and Stochastic Analysis*, Article ID 81508, 1-22.

Yadavalli, V.S.S., Kathiresan, J. and Anbazhagan, N., 2018. A Continuous Review Inventory System with Retrial Customers and Two-Stage Service. *Applied Mathematics and Information Sciences*, 12(2), 441-449.

Yadavalli, V.S.S., Anbazhagan, N. and Jeganathan, K., 2015. A Retrial Inventory System with Impatient Customers. *Applied Mathematics and Information Sciences*, 9(2), 637-650.