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Corrective Measures in Linear Regression Model Plagued with Heteroscedasticity: A Monte Carlo Approach

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Abstract. . In the presence of heteroscedasticity, Ordinary Least Squares (OLS) estimators remain unbiased but no longer efficient. The study examined the corrective measures in linear regression model plagued with heteroscedasticity. Four different heteroscedasticity test were examined of which Goldfeld Quandt test perform better when the sample size is small, while Glejser test is appropriate for larger sample sizes. The HC3 results in better inference for small samples and performed equally with other HCCM for large samples. The performance of OLS, WLS and HC3 compared shows that WLS estimator is preferable in parameter estimation if the model is plagued with heteroscedasticity with known functional form.

Key words: heteroscedasticity consistent covariance Matrix; Goldfeld-Quandt test; Glejser test; Breusch Pagan test; White test and weighted Least Squares
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Résumé (Abtsract in French) Cet article se focalise sur le modèle linéaire multivarié des moindres carrés ou ordinaire (MLMO) en presence d'hétéroscédasticité. Dans un tel cas, les estimateurs sont sans biais certes mais ils sont non-efficacces. Quatre tests d'hétéroscédasticité ont été appliqués. Le test de Goldfeld-Quandt est plus performant pour de petites tailles alors que le test Glejser est meilleur pour de grandes tailles. Les results HC3 sont meilleurs que ceux relatifs à HCCM pour de petites tailles mais leur sont équivalent pour de grandes tailles au niveau de la qualité de l'inférence. Les performances comparées des modèles MLMO, WLS et HC3 montrent que les estimateurs WLS sont préférables par rapport à l'estimation des paramètres en presence d'hétéroscédasticité lorsque les formes fonctionelles ne sont pas connues.

1. Introduction

Linear regression models requires the underlying assumption that the variance of the errors given the independent variable is constant. Heteroscedasticity is a violation of this assumption, it occurs when the variance of the errors across the observations are heterogenous.

Heteroscedasticity does not affect the properties of unbiasedness and consistency of the OLS, but the minimum variance of the estimators will be affected, which can invalidate inference (White, 1980). Klein *et.al* (2016) proposed a simple measure of heteroscedasticity which does not need a parametric model and is able to detect omitted non linear terms. The measure utilizes the dispersion of the squared regression residuals. Box-Cox transformation was applied to economic data as a corrective measure for heteroscedasticity. The techniques introduced the geometric mean into the transformation by including the Jacobian of rescaled power transformation with likelihood (see (Nwakuya and Nwabueze, 2018)). Breusch and Pagan (1979) and Cook and Weisberg (1983) proposed a Breusch-Pagan test which test the null hypothesis that the residual variances are unrelated to a set of explanatory variables.

The squared OLS residuals regressed on all predictors, cross products, squares of predictors and the intercept was adopted by (White, 1980) commonly known as White test. In the context of structural equation model, Klein and Schermelleh-Engel (2010) proposed Z_{het} statistic that detected heteroscedasticity caused by omitted predictors in structural equation models.

In this paper, we examined the corrective measures in linear regression model plagued with heteroscedasticity, using Monte carlo approach. The rest of the paper is organised as follows: In Section 2, the OLS estimator, test for detecting heteroscedasticity and method of parameter estimation adopted are presented, the Monte Carlo experiment carried out in the work is discussed in Section 2.4. Results are presented in Section 3, discussion of the results in Section 4 , while Section 5 provides some concluding remarks.

2. Material and Methods

2.1. The Model

Given that the linear regression model for jth observations can be written as:

$$y_j = \beta_0 + \beta_1 X_{1j} + \beta_2 X_{2j} + \cdots + \beta_q X_{qj} + \varepsilon_j \tag{1}$$

where y$_j$ denote the response variable for the jth observations, X's are independent variables, $\beta_1, \cdots, \beta_q$ are model parameters and ε_j is the departure term. The model in ((1)) in matrix form is:

$$Y = X\beta + \varepsilon \tag{2}$$

where;

$$Y = \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_m \end{bmatrix} = \begin{bmatrix} 1 & X_{11} & X_{21} & \cdots & X_{m1} \\ 1 & X_{12} & X_{22} & \cdots & X_{m2} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 1 & X_{1q} & X_{2q} & \cdots & X_{mq} \end{bmatrix} \begin{bmatrix} \beta_0 \\ \beta_1 \\ \vdots \\ \beta_q \end{bmatrix} + \begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \vdots \\ \varepsilon_m \end{bmatrix} \tag{3}$$

where Y is $m\times$ 1 response variables, X is m $\times$ q matrix of explanatory variables, β is q $\times$ 1 vector of parameters and ε is m $\times$ 1 vector of departure term. Assume, there is presence of heteroscedasticity in (2). Then,

$$E(\varepsilon_j^2) = \sigma_j^2 \tag{4}$$

2.2. Tests for Detecting Heteroscedasticity

Before examining the corrective procedures for parameter estimate of (2), we have to find out the presence of heteroscedasticity. To investigate this violation, formal test were carried out. The tests for detecting heteroscedasticity considered in this study are:

2.2.1. Glejser Test

This test is based on the assumption that σ_j^2 is influenced by one variable Z that is, there is only one variable which is influencing heteroscedasticity. This variable could be either one of the explanatory variables or it can be chosen from some extraneous sources. The test procedure is as follows:

1. Use OLS and obtain the residual vector *e* on the basis of available study and explanatory variables.
2. Choose Z and apply OLS to $|e_j|$ = Υ_0+ $\Upsilon_1 Z_j^h$ + v$_j$, where v$_j$ is the associated disturbance term, Υ_0 and Υ_1 are associated parameters.
3. Test H$_0$: Υ_1= 0 using t-ratio test statistic.
4. Conduct the test for h = $\pm 1, \pm\frac{1}{2}$. So the test procedure is repeated four times.

2.2.2. Goldfeld-Quandt Test

This is based on the assumption that σ_j^2 is positively related to X_{ij}, that is, one of the explanatory variables explain the heteroscedasticity in the model. Let j^{th} explanatory variable explain the heteroscedasticity, so $\sigma_j^2 \propto X_{ij}$ or $\sigma_j^2 = \sigma^2 X_{ij}$. The test procedure is as follows:

1. Rank the observations according to the decreasing order of X_j.
2. Split the observations into two equal parts leaving q observations in the middle. So that each part contains $\frac{n-q}{2}$ observations. Where $q = \frac{n}{4}$
3. Run two separate regressions in the two parts using OLS and obtain the residual sum of squares SS_{res1} and SS_{res2}
4. The test statistic is:

$$F_0 = \frac{SS_{res2}}{SS_{res1}} \tag{5}$$

2.2.3. Breusch-Pagan Test

This test can be applied when the replicated data is not available but only single observations are available. When it is suspected that the variance is some functions of more than one explanatory variable, then Breusch Pagan test can be used. The test procedure is as follows:
Assume that,

$$\sigma_j^2 = \gamma_1 + \gamma_2 Z_{j2} + \cdots + \gamma_p Z_{jp} \tag{6}$$

1. Ignoring heteroscedasticity, apply OLS to $y_j = \beta_0 + \beta_1 X_{1j} + \cdots + \varepsilon_j$ and obtain residual $e = y - X\beta$.
2. Construct the variables:

$$g_j = \frac{e_j^2}{(\sum_{j=1}^{n} e_j^2/n)} = \frac{ne_j^2}{SS_{res}} \tag{7}$$

   where SS_{res} is the residual sum of squares based on e_j's.
3. Run regression of g on $Z_1, Z_2, \cdots, Z_p$ and get residual sum of squares SS^*_{res}.
4. Calculate the test statistic:

$$Q = \frac{1}{2}(\sum_{j=1}^{n} g_j^2 - SS^*_{res}) \tag{8}$$

   which is asymptotically distributed as χ^2 distribution with (p - 1) degrees of freedom.

2.2.4. White Test

This test is very similar to that of the Breusch-Pagan test. The test is based on the residual of the fitted model.

2.3. Methods of Estimating Parameters in the Presence of Heteroscedasticity

Ordinary Least Squares (OLS) estimators are no longer efficient in the presence of heteroscedasticity. The remedial measures involves obtaining unbiased estimators that are efficient, unbiased and consistent. The method for estimating parameters in the presence of heteroscedasticity considered in this study is given by:

2.3.1. Weighted Least Squares (WLS)

This method involves transforming the original variables such that the transformed variables satisfies the assumptions of the classical linear model.
Given a linear model:

$$y_j = \beta_0 + \beta_1 X_{1j} + \varepsilon_j \tag{9}$$

Assuming that the heteroscedastic variances σ_j^2 is known. Divide (9) through by σ_j, we obtain:

$$\frac{y_j}{\sigma_j} = \beta_0 \frac{1}{\sigma_j} + \beta_1 \frac{X_j}{\sigma_j} + \frac{\varepsilon_j}{\sigma_j} \tag{10}$$

That is;

$$y_j' = \beta_0' + \beta_1 X' + U_j \tag{11}$$

where $y_j' = \frac{y_j}{\sigma_j}$, $\beta_0' = \beta_0 \frac{1}{\sigma_j}$, $X' = \frac{X_j}{\sigma_j}$ and $U_j = \frac{\varepsilon_j}{\sigma_j}$.

The variance of the transformed disturbance term in (11) is homoscedastic. Since the transformed model in (11) is homoscedastic, then applying OLS to the transformed model will produce estimators that are Best Linear Unbiased Estimator (BLUE). The WLS estimator of the parameters is given by:

$$\hat{\beta} = (X'WX)^{-1} X'WY \tag{12}$$

where W is a m $\times$ m diagonal matrix with weights as diagonal elements.

2.3.2. Heteroscedasticity Consistent Covariance Matrix (HCCM)

The exact form of heteroscedasticity might be unknown in most cases, where WLS approach might be inappropriate. In such cases, HCCM approach would be adopted to improve inference.
Given the variance of β as:

$$Var(\hat{\beta}) = (X'X)^{-1} X'\Phi X (X'X)^{-1} \tag{13}$$

where Φ is an m $\times$ m diagonal matrix with diagonal element Var (ε_i), when Φ is unknown, we need a consistent estimator of Φ which can obtained by applying HCCM. In this method β is estimated using OLS, then HCCM is used to estimate the standard errors of the estimates.

The basic idea behind the HCCM estimator is to use e_j^2 to estimate ϕ_{jj}, where e_j = y - Xβ. This can be thought as a result of estimating the variance of ε_j with a single observation:

$$\phi_{jj} = (e_j - 0)^2/1 = e_j^2 \tag{14}$$

Then, let Φ = diag[e_j^2], which gives:

$$HC0 = (X'X)'X'\Phi X(X'X)^{-1} = (X'X)^{-1}X'diag(e_j^2)X(X'X)^{-1} \tag{15}$$

HC0 is a consistent estimator of Var (β) in the presence of heteroscedasticity of an unknown form but results to incorrect inference when the sample size is small (White, 1980). (Mackinnon and White, 1985) considered three alternative estimators to improve the small sample properties of HC0. These includes:

$$HC1 = \frac{N}{N-P}(X'X)^{-1}X'diag(e_j^2)X(X'X)^{-1} = \frac{N}{N-P}HC0 \tag{16}$$

HC1 makes a degree of freedom correction that inflates each residual by a factor of $\sqrt{\frac{N}{N-P}}$, where N is the sample size and P is the number of parameters.

$$HC2 = (X'X)^{-1}X'diag(\frac{e_j^2}{1-h_{jj}})X(X'X)^{-1} \tag{17}$$

HC2 is based on the idea that $\frac{e_j^2}{1-h_{jj}}$ will be a less biased estimator of σ_j^2 from Var (ε_j) = σ^2(1 - h_{jj}), where, h_{jj} = $X_j(X'X)^{-1}X_j'$.

$$HC3 = (X'X)^{-1}X'diag(\frac{e_j^2}{(1-h_{jj})^2})X(X'X)^{-1} \tag{18}$$

HC3 uses another estimator of σ_j^2 that further inflates e_j^2. Since 0 $\leq h_{jj} \prec 1$, dividing ε_j^2 by (1 - h_{jj})^2. The variations of HCCM estimators are easy to program since they are functions of statistics routinely computed by standard regression packages.

2.4. Data Generating Process

Monte Carlo experiments were carried out to examine the small sample behaviour of tests using the OLS, WLS and the four variations of the HCCM. Each simulation carried out was based on (2). The experiment procedure are as follows:

Step 1: Assumed numeric values were assigned to the parameters β_0 and β_1. That is:

$$y_j = 1.5 + 2.0X_{1j} + \varepsilon_j \tag{19}$$

Step 2: The departure term were generated from normal distribution (0, 1).
Step 3: The explanatory variables were generated from uniform distribution [0,

10].
Step 4: For sample sizes n = 25, 50, 100, 250 and 500, the experiment was replicated 1000 times in turn.
Step 5: Heteroscedasticity was introduced such that h(x) = $\sigma^2 X^\delta$, where δ takes the values 0.5 and 1 (mild), 2 (moderate) and 4 (severe)

Step 6: For each Monte Carlo trial, data set were generated in which the power of the various tests for heteroscedasticity were calculated. The R-software package was used to investigate this.

3. Results

The power of test was examined on Breusch-Pagan, Glejser, Goldfeld-Quandt and White test. The Monte Carlo experiment when the sample sizes n = 25, 50, 100, 250 and 500 are presented in the tables below. HC3 is preferred to HC0, HC1 and HC2 because it results in better inference when the sample is small.
Other estimation methods such as OLS and WLS (misspecified functional form) were also examined using the true and estimated standard errors at each sample size.

Table 1 presented the power of Breusch-Pagan, Glejser, Goldfeld-Quandt and White test for different degrees of heteroscedasticity ranging from mild to severe. As the sample size increases the power of each tests increases but when the heteroscedasticity tends to be severe, the test maintain a constant value as the sample size increases from 100 to 500. For small sample size, White test consistently has low power of test compared to the other tests for all degrees of heteroscedasticity. Goldfeld-Quandt has higher power of test compared to all other tests when the sample size is small. Figure 3 presents the power curve for the four tests of heteroscedasticity with the form of heteroscedasticity considered in the study. The figure reflects the key results that are shown in table 1. Figure 3 as the sample sizes and the degree of heteroscedasticity increases the power of the tests also increases. Goldfeld-Quandt have higher power of test when the sample size is small but Glejser test have slightly higher power of test as the sample size increases. The White test consistently have lower power compared to the other test.

Table 2 presents the comparison of the true standard error and estimated standard error of the variations of HCCMEs when the form of heteroscedasticity is moderate (h(x) = X^2). At n = 50, 100, the estimated standard error of the HC3 tends to be closer to the exact standard error compared to other HCCME variations. Figure 2 shows that HC3 results in better inference when the sample size is small than HC0, HC1 and HC2. The bias of the estimated slope parameter is shown for all estimation methods in figure 3, which shows that the estimation methods result in unbiased estimate, the larger bias that can be seen(δ = 4) can be explained by the increase in variance of the slope estimates rather than a biased estimator. In figure 3, the WLS estimates were shown to has the lower variance than OLS estimates. As the degree of heteroscedasticity increases, it was shown that the variance of the OLS estimate increases, while the WLS estimates shows more stability even with increase in level of heteroscedasticity.

Table 3 shows the comparison between the average of the estimated standard error and the true standard deviation of the slope estimates. It was observed that when there is homoscedasticity in the model ($h(x) = X^0$), the WLS and the OLS estimators are the same. In the presence of heteroscedasticity, it was observed that the WLS has the least standard error compared to the other estimation method considered.

The incorrect variance function of WLS is inefficient since it's standard error is higher than the WLS of the correct variance. The standard error value of the HC3 are closer to the true standard errors for all levels of heteroscedasticity compared to OLS standard error.

Comparing the type 1 error rate (empirical significance level) with the nominal significance level used in carrying out the test (0.05). Table 4 shows that for all degrees of heteroscedasticity, the type 1 error rate obtained using OLS, WLS and HC3 shows lesser deviation from the nominal significance level than the WLS with misspecified variance function.

As the degree of heteroscedasticity increases, the deviation value of OLS and incorrect WLS estimators show higher deviation from the nominal significance level. Figure 3 shows that the OLS results in high type 1 error rate as the form of heteroscedasticity increases with increase in sample sizes. It was observed that using WLS with misspecified variance function results in higher type 1 error rate for all degrees of heteroscedasticity. The figure also shows that WLS and HC3 performed very well for all levels of heteroscedasticity.

Table 1. Results of Power of Test for Heteroscedasticity

Method	**Breusch-Pagan Test**				**Glejser Test**			
Sample Size/Error Struture	$\mathbf{X}^{\frac{1}{2}}$	$\mathbf{X}$	$\mathbf{X}^2$	$\mathbf{X}^4$	$\mathbf{X}^{\frac{1}{2}}$	$\mathbf{X}$	$\mathbf{X}^2$	$\mathbf{X}^4$
25	0.156	0.366	0.692	0.910	0.213	0.490	0.849	0.980
50	0.381	0.764	0.977	0.999	0.516	0.929	0.999	1.000
100	0.686	0.993	1.000	1.000	0.843	1.000	1.000	1.000
250	0.968	1.000	1.000	1.000	0.994	1.000	1.000	1.000
500	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	Goldfield-Quandt Test				**White Test**			
25	0.259	0.531	0.903	0.999	0.088	0.190	0.428	0.744
50	0.525	0.909	0.999	1.000	0.228	0.610	0.917	0.988
100	0.802	0.998	1.000	1.000	0.605	0.984	1.000	1.000
250	0.979	1.000	1.000	1.000	0.972	1.000	1.000	1.000
500	0.999	1.000	1.000	1.000	1.000	1.000	1.000	1.000

Table 2. Comparison of the Estimated and True Standard Error of the Variations of HCCMEs in the Presence of Heteroscedasticity of the form h (x) = X^2

Method	n = 25		n = 50		n = 100		n = 250		n = 500	
	True SE	Est. SE	True SE	Est. SE	True SE	Est. SE	True SE	Est. SE	True SE	Est. SE
HC0	0.456	0.0408	0.312	0.289	0.209	0.204	0.138	0.136	0.096	0.096
HC1	0.456	0.0425	0.312	0.295	0.209	0.206	0.138	0.136	0.096	0.096
HC2	0.456	0.0433	0.312	0.297	0.209	0.207	0.138	0.137	0.096	0.096
HC3	0.456	0.0459	0.312	0.306	0.209	0.210	0.138	0.137	0.096	0.096

Table 3. Comparison of the Estimated and True Standard Error of the Estimators for n = 250

Method	h(x) = X^0		h(x) = $X^{\frac{1}{2}}$		h(x) = X^4	
	True SE	Est. SE	True SE	Est. SE	True SE	Est. SE
OLS	0.0215	0.0219	0.0305	0.0326	1.2396	1.0703
Incorrect WLS	0.0217	0.0218	0.0261	0.0261	0.2802	0.3896
WLS	0.0215	0.0219	0.0253	0.0259	0.0411	0.0408
HC3	0.0215	0.0220	0.0305	0.0309	1.2396	1.2263

Table 4. Deviation of Type 1 Error Rate from the Nominal Significance Level

Heteroscedasticity	Method	Sample Size				
		n = 25	n = 50	n = 100	n = 250	n = 500
$X^{\frac{1}{2}}$	OLS	-0.010	-0.015	-0.010	-0.010	-0.009
	Incorrect WLS	0.011	0.017	0.004	-0.002	-0.001
	WLS	-0.015	0.002	-0.012	-0.004	-0.007
	HC3	-0.005	-0.004	-0.004	-0.001	0.002
X	OLS	-0.001	-0.008	-0.005	-0.012	-0.006
	Incorrect WLS	0.008	0.011	0.003	-0.003	-0.008
	WLS	-0.003	0.003	0.000	-0.005	-0.008
	HC3	-0.003	-0.007	-0.003	0.001	0.004
X^2	OLS	0.019	0.019	0.014	0.005	0.013
	Incorrect WLS	0.042	0.048	0.039	0.017	0.022
	WLS	-0.001	-0.002	0.005	-0.007	0.000
	HC3	0.003	-0.002	0.003	0.002	0.006
X^4	OLS	0.063	0.071	0.067	0.039	0.041
	Incorrect WLS	0.080	0.092	0.073	0.032	0.020
	WLS	0.014	-0.007	-0.004	0.010	0.005
	HC3	0.009	0.005	0.008	0.004	0.002

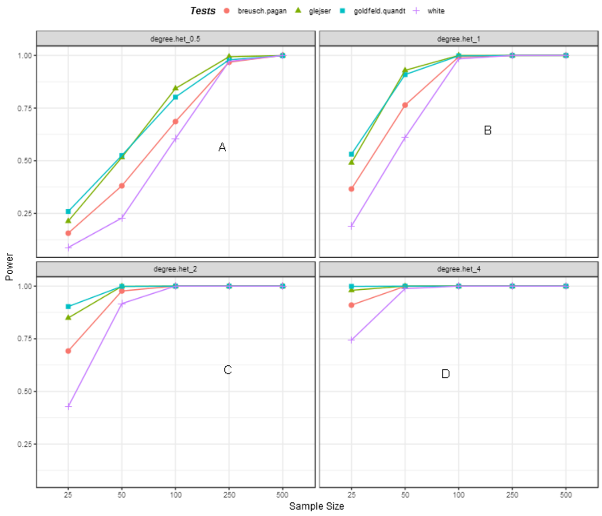

Figure 1: Power curve of tests for heteroscedasticity. **(A) h(x) =** $\mathbf{X}^{\frac{1}{2}}$, **(B) h(x) = X**, **(C) h(x) =** $\mathbf{X}^{2}$, **(D) h(x) =** $\mathbf{X}^{4}$. Note: h(x) stands for the form of heteroscedasticity

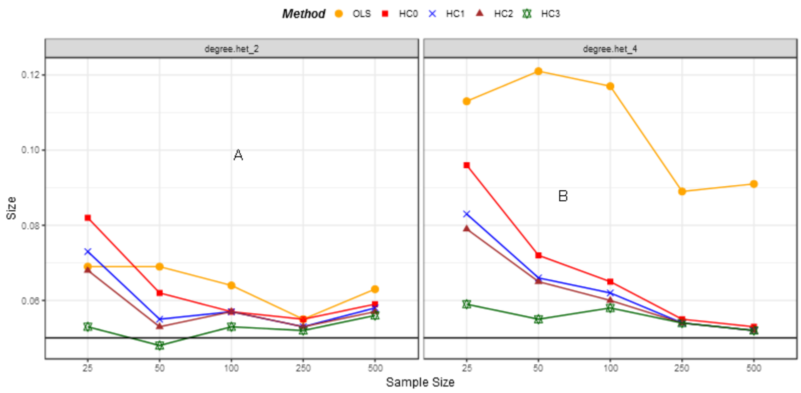

Figure 2: Comparing type 1 error rate of estimating the slope parameter for different variations of HCCME for different degree of heteroscedasticity. **(A) h(x) =** $\mathbf{X}^{2}$, **(B) h(x) =** $\mathbf{X}^{4}$. Note: h(x) stands for the form of heteroscedasticity

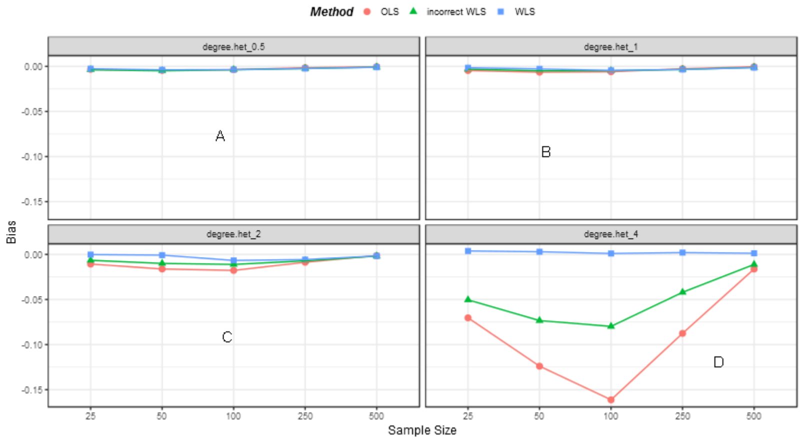

Figure 3: Biased of the estimated slope parameter. **(A) h(x) =** $\mathbf{X}^{\frac{1}{2}}$, **(B) h(x) = X**, **(C) h(x) =** $\mathbf{X}^2$, **(D) h(x) =** $\mathbf{X}^4$. Note: h(x) stands for the form of heteroscedasticity.

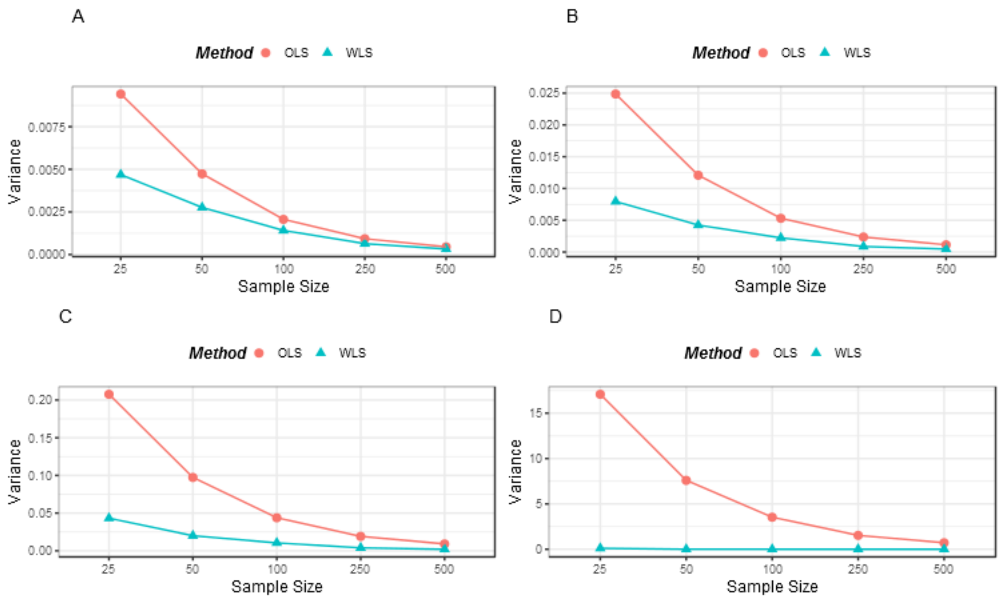

Figure 4: Variance of the estimated slope parameter. **(A) h(x) =** $\mathbf{X}^{\frac{1}{2}}$, **(B) h(x) = X**, **(C) h(x) =** $\mathbf{X}^2$, **(D) h(x) =** $\mathbf{X}^4$. Note: h(x) stands for the form of heteroscedasticity.

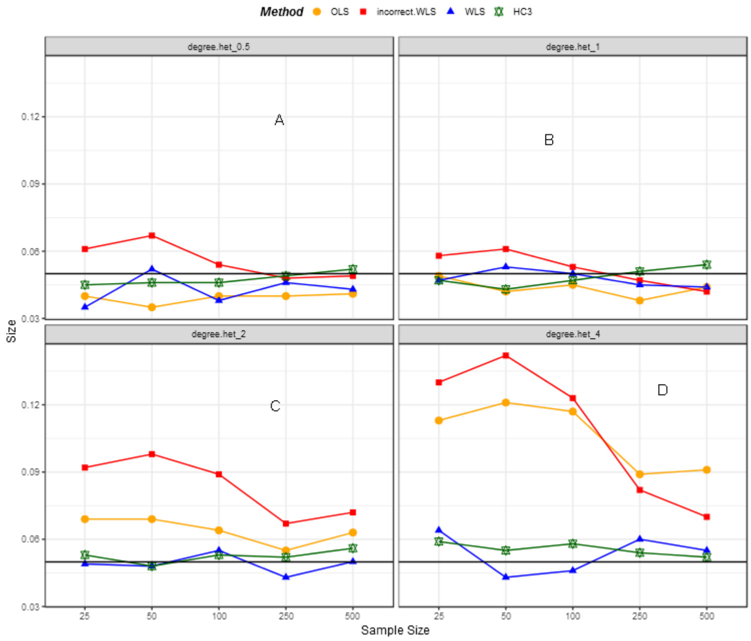

Figure 5: Type 1 error rate of estimating the slope parameter for different degrees of heteroscedasticity. **(A)** $h(x) = X^{\frac{1}{2}}$, **(B)** $h(x) = X$, **(C)** $h(x) = X^2$, **(D)** $h(x) = X^4$. Note: h(x) stands for the form of heteroscedasticity.

4. Discussion of Results

Heteroscedasticity affects the performance of OLS estimators, this study examined and compared the power of the various tests for detecting heteroscedasticity and compared the methods of estimating the slope parameter of the regression model in the presence of heteroscedasticity. It was shown that when the regression model has non constant error variance, the OLS estimators would be unbiased but no longer efficient (Goldberger, 1964). The standard error of the estimators are incorrectly calculated by OLS, leading to invalid inference in hypothesis testing.

White test has a lower power of test in detecting heteroscedasticity compared to the other tests considered, while Goldfeld-Quandt has a higher power of test compared to all other tests when the sample size is small (Ruud, 2000). The results indicated that the WLS is unbiased and efficient when the variance function is known or could be estimated correctly from the data, but when the variance is of incorrect specification, the WLS might be worse than OLS estimator.

When the form of heteroscedasticity is unknown resulting to inconsistency of the OLS estimators, results above indicated that the HC3 provides a consistent estimator of the standard error of the parameter estimates (White, 1980).

5. Conclusion

Goldfeld-Quandt test is appropriate in detecting the presence of heteroscedasticity when the sample size is small ($n \leq 50$), while Glejser test for large sample size.

If the form of heteroscedasticity is known, Weighted Least Squares (WLS) is preferred in estimating parameters since the resulting estimates are BLUE. However, in real life situations, it is not always possible to know the form of heteroscedasticity.

Using OLS in the presence of heteroscedasticity is inconsistent, we can therefore use HC3 to get the consistent standard errors of our estimates which avoid the adverse effects of heteroscedasticity on hypothesis testing when the form of heteroscedasticity is unknown.

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