AJAS / SPAS

ISSN 2316-0861

African Journal of Applied Statistics
Vol. 7 (1), 2020, pages 805 –827.
DOI: http://dx.doi.org/10.16929/ajas/2020.805.243

ISSN 2316-0861

Modelling exchange rate volatility in The Gambia using dynamic conditional correlation model

Fatoumata Baboucar Omar Kah (1,*) **and Abdou Ka Diongue** (2)

(1) Fatoumata Baboucar Omar Kah, Masters in Applied Mathematics
Statistician, Planning Directorate
Ministry of Basic and Secondary Education, The Gambia

(2) Abdou Ka Diongue, PhD of Applied Mathematics
LERSTAD, Department of Applied Mathematics
University Gaston Berger of Saint-Louis, Senegal

Received November 11, 2019; Accepted February 10, 2020; Published online May 24, 2020

Abstract. (Short abstract) The relationship between different international stock markets is of importance for both financial practitioners and academicians in order to manage risks. Especially after the financial crisis, the pronounced financial contagion draws the public attention to look into such associations. However, measuring and modelling dependence structure becomes complicated when asset returns present non-linear, non-Gaussian and dynamic features. This paper examines the time-varying conditional correlations to the weekly exchange rate returns for the USD, EURO and GBP against The Gambian Dalasi (GMD) during the period 2000 to 2017. (to be contunued in page 806)

Key words: Exchange rate; returns; volatility; DCC-GARCH; EGARCH; GJR-GARCH
AMS 2010 Mathematics Subject Classification Objects : 62P20; 62M10; 62M20.

---

*Fatoumata Baboucar Omar Kah: fatoumata.kah@gmail.com
Abdou Ka Diongue: abdou.diongue@ugb.edu.sn

Full Abstract The relationship between different international stock markets is of importance for both financial practitioners and academicians in order to manage risks. Especially after the financial crisis, the pronounced financial contagion draws the public attention to look into such associations. However, measuring and modelling dependence structure becomes complicated when asset returns present non-linear, non-Gaussian and dynamic features. This paper examines the time-varying conditional correlations to the weekly exchange rate returns for the USD, EURO and GBP against the Gambian Dalasi (GMD) during the period 2000 to 2017. We use a dynamic conditional correlation (DCC) multivariate GARCH model. This model can be simplified by estimating univariate GARCH models for each return series, and then, using transformed residuals resulting from the first stage, estimating a conditional correlation estimator. DCC-GARCH model was implemented for two different assumptions of the error distribution; assuming Gaussian and Student t-distribution. Empirical results show substantial evidence of significant increase in conditional correlation. It is also clean that, the Student t-distributed errors better forecast the conditional correlation.

Résumé: (French) L'évaluation des interactions entre différents marchés financiers est d'une importance particulière en gestion de risque aussi bien pour les chercheurs que pour les praticiens de la finance de marché. En effet, après une crise financière, à cause de l'effet contagion, cette association est particulièrement étudiée. Cependant, la non-linéarité et les queues épaisses généralement observées sur les séries de rendements financiers rendent la modélisation de cette structure de dépendance très compliquée. Dans ce papier, nous examinerons les corrélations conditionnelles dynamiques sur les rendements de taux de change hebdomadaires USD/Gambian Dalasi (GMD), EURO/GMD, et GBP/GMD sur la période allant de 2000 à 2017. Nous utiliserons un modèle GARCH multivarié à corrélations conditionnelles dynamiques. L'estimation du modèle peut se faire en deux étapes : d'abord estimer un modèle GARCH univarié sur chaque série, puis estimer la corrélation conditionnelle sur les résidus. Une application du modèle avec bruit Gaussien et Student-t est considérée. Les résultats ont mis en évidence une significativité substantielle de la corrélation conditionnelle. En terme de prévision, le modèle avec bruit student-t est plus performant.

The authors.

(1) **Miss Fatoumata Baboucar Omar Kah**, Ms.c, is Statistician at the Planning Directorate, Ministry of Basic and Secondary Education, The Gambia. She workded under the supervison for her Ms.c. dissertation.

(2) **Abdou Ka Diongue**, Ph.D., is full professor at Department of Applied Mathematics, University Gaston Berger of Saint-Louis, Senegal.

1. Introduction

Volatility is a concept that helps us measure the uncertainty of a particular market or financial value (stocks, bonds, currencies, commodities, etc.) In other words, one can say that the volatility in finance is a measure of fear or instability of returns Abdalla and Zakaria (2012). From the point of view of any investor, when it is said that the price of a stock or exchange rate is very volatile, it often refers to as "violent" fluctuations outside what is commonly recorded. It is widely known that volatility varies over time and tends to cluster in periods of large volatility and periods of tranquility (May and Farrell (2017)). This phenomenon is what we called heteroskedasticity.

Over the last few decades, exchange rate movements and fluctuations have become an important subject of macroeconomic analysis and have received a great deal of interest from academics, financial economists and policy makers, particularly after the collapse of the 1994 Bretton Woods agreement of fixed exchange rates among major industrial countries. Since then, there has been an extensive debate about the topic of exchange rate volatility and its potential influence on welfare, inflation, international trade and degree of external sector competitiveness of the economy and also its role in security valuation, investment analysis, profitability and risk management (Bosnjak et al. (2016)). Consequently, a number of models have been developed in empirical finance literature to investigate this volatility across different regions and countries. Well known and frequently applied models to estimate exchange rate volatility are the autoregressive conditional heteroskedastic (ARCH) model by Engle (1982) and generalized (GARCH) model developed by Bollerslev (1986).

Empirical researches have been carried out in the area of estimating and forecasting using GARCH models. Sengupta and Sfeir (1996) studied the exchange rate volatility of the US dollar against the British pound, the yen, the German mark, and the French franc; Baillie et al. (1996) analyzed the volatility of the exchange rate of the German mark against the dollar; Tse (1998) studied the volatility of the yen-dollar exchange rate. More recent work has also followed this line of modeling: Longmore and Robinson (2004) analyzed the behavior of the Jamaican dollar against the US dollar. Sandoval (2006) analyzed the exchange rate volatility of the currencies of some emerging financial markets. Siddiqui (2009) analyzed the volatility of the exchange rate of the Pakistani rupee in relation to the currencies of Canada, the United States, Japan, United Kingdom and the European Monetary Union. Sarno et al. (2004) and McMillan and Speight (2010) studied the dynamics of exchange rates intraday data. Bauwens and Sucarrat (2010) studied the volatility of the Norwegian krone; Olowe (2009) analysed the exchange rate volatility of the Nigerian currency against the US dollar. Other scholars have chosen to analyze the exchange rate volatility using stochastic volatility models. MacDonald and Taylor (1994) studied the daily exchange rate of the German mark in respect of applying various stochastic volatility models. Mahieu and Schotman (1998) analyzed the volatility of bilateral exchange rates of the currencies of US, Japan, Germany, and the United Kingdom. Taylor (1986) uses a stochastic volatility model to study the volatility of the exchange rate of sterling against the US dollar. Arranz and Iglesias (2005) analyzed the behavior of the peso-dollar, compared with the exchange rates of Venezuela, South Africa, Norway and the UK, using the stochastic volatility model proposed by Taylor (1986). Abdalla and Zakaria (2012) estimated from a Panel of nineteen Arab countries occurrence or nonoccurence and degree of asymmetry of the exchange rate series which provide evidence of leverage effects for the majority of currencies. Peters (2008) evaluated how forecasts of the Dynamic Conditional Correlation model of Engle and Sheppard (2001) performs compared to the traditional one (sample covariance). His results shown that the dynamic conditional correlation tend to out perform the covariance matrix based on historical data in the short run, while in the long run the reverse relationship holds. Hartman and Sedlak (2013) examined the performance of two multivariate GARCH models BEKK and DCC, applied on ten years exchange rates data. Their results indicated that the BEKK model performs relatively better than the DCC model, and both these models perform better than the univariate GARCH(1,1) model. May and Farrell (2017) examine the key nominal exchange rates of the South African rand using various GARCH models. Their results provide evidence of leverage effects, indicating that negative shocks imply a higher next period volatility than positive shocks.

To our knowledge, (Marreh et al. (2014)) modeling of exchange rate is the only work done on modeling exchange rate volatility in the Gambian market and thus helping to fill the knowledge gap. In their study, they model the USD and EURO against GMD using ARMA-GARCH models. Their empirical results revealed that the distribution of the return series was heavy-tailed and volatility was highly persistent in The Gambian foreign exchange market.

The objective of this study is to investigate the volatility and conditional relationship of exchange rates in The Gambia and to construct a model using a multivariate DCC-GARCH model.The DCC-GARCH is mainly a generalization of the CCC-GARCH model developed by (Bollerslev (1990)). This model, to the best of our knowledge has not been used in The Gambian context of exchange rate. This model estimates the parameters in two steps that makes it relatively easy to use in practice. The first step uses the univariate GARCH model to estimate the conditional variance for each asset. In the second step, the parameters of the conditional correlation given the parameters of the first step are estimated. In this study the implementation of the DCC-GARCH model will be considered, using Gaussian and Student distributed errors. This study contributes in two ways. First, it will make a significant contribution to the knowledge base on the selected topic in the context of developing countries, since there is little work done on modeling exchange rate volatility in The Gambian market and thus helping to fill the knowledge gap. Second, this study applied the DCC on GARCH model and different GARCH extensions which is quite different from what has been done in the previous studies.

The outline of this study is as follows: Section 2 describes the exchange rate data from the Central Bank of The Gambia. In Section 3, we present the two models namely the Univariate GARCH model and the multivariate GARCH model (DCC-GARCH) while Section 4 deals with applications of the data, and Chapter 5 gives the conclusion of our findings.

2. Data description

The data used for performing the empirical analysis is weekly data of the exchange rates of The Gambia from 2000 to 2017. The data is obtained from the Central Bank of The Gambia (CBG). The currencies used in this study are US Dollar (USD), EURO and the Great British Pound (GBP) exchange rates against The Gambian Dalasi (GMD). As in most empirical finance literature, the variable to be modeled is percentage daily exchange rate returns which is the first difference of the natural logarithm(ln) of the exchange rate and is given by the following equation:

$$r_t = \ln(e_t/e_{t-1}) * 100.$$

where r_t is the daily percentage return to the exchange rate and e_t and e_{t-1} denote the exchange rate at the spot day and previous day respectively.

The sample consists of 936 observations for each exchange rate return against the Dalasi. R-Software is used in this empirical study as the statistical software for the data analysis.

From Table 1, all the returns shows evidence of non normality with negative skewness, means that all the three exchange rate returns are skewed to the left. The mean return for the three currencies are close to zero. The kurtosis are large for all the three currencies, excess kurtosis is found in the summary above which supports an appropriateness of the usage of Student-t distribution. The weekly standard deviation (SD) shows that the GBP return is the most volatile with a value of (2.02), followed by the EURO return and USD return respectively. All the currencies show evident of fat tails since their kurtosis exceed 3 while negative skewness for the currencies indicate that left tail are particularly extreme.

To test for normality of the skewness and kurtosis, Jarque-Bera test was used and the main focus is the p-values for the chi square which all are (0.0000). Since the p-values are all less than 5%, it indicates that there is enough evidence to reject normality of the return series (H_0). In addition, the empirical distribution present in figure 1, confirm the fact that the difference currencies are not normally distributed.

Prior modeling the exchange rate return series, Augmented Dickey fuller test was employed to determine the stationarity of the returns of the exchange rate series. Testing the null hypothesis of a unit root against the alternative hypothesis of

Table 1. Summary Statistics

	USD	EURO	GBP
A. Summary of the exchange rate returns			
Minimum	-25.99	-26.60	-33.38
Median	0.07	0.12	0.13
Mean	0.15	0.17	0.13
Maximum	14.91	8.48	12.30
SD	1.90	1.94	2.02
Kurtosis	68.91	49.85	92.13
Skewness	-4.16	-3.59	-5.60
B. Jarque Bera Test			
	172110	87625	314610
	(0.000)	(0.000)	(0.000)
C. Augmented Dickey Fuller (ADF) Test			
t-Statistic	-9.5514	-9.0963	-9.9845
p-value	0.01	0.01	0.01
lag order	9	9	9
D. Ljung Box Test			
Q-Statistic	15.515	8.6711	14.766
p-value	0.1144	0.5636	0.1408
df	10	10	10
E. ARCH-LM Diagnostic Test			
LM Statistic	4706	10293	20569
p-value	0	0	0
lag order	0	0	0

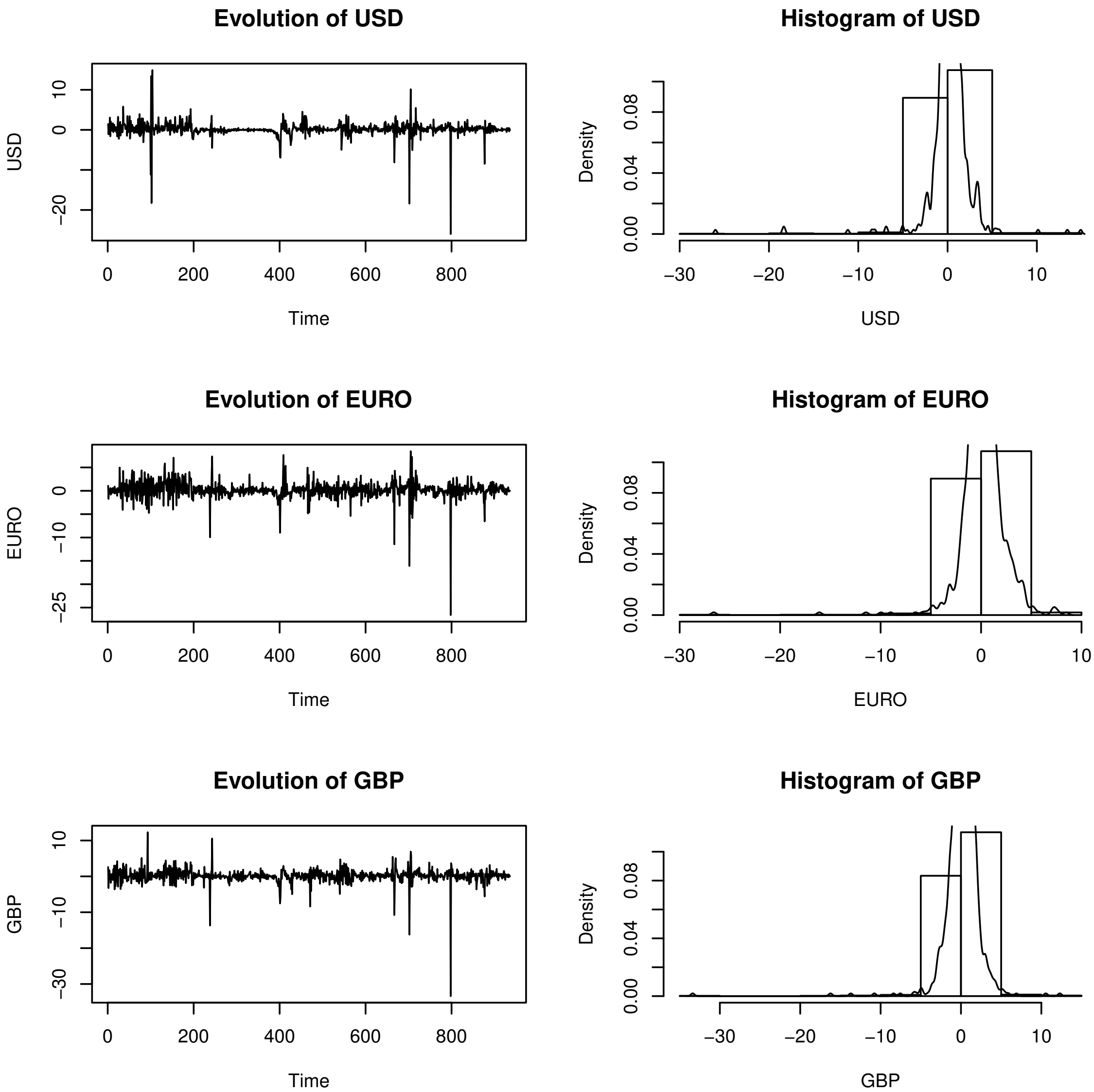
no unit root for the three return series. Since the p-values for the variables are all less than 5%, we reject the null hypothesis and accept the alternative. This means that the exchange rate return series are all stationary thus, there is no differencing of the return series.

Ljung Box Test was also employed to check whether the residuals are random, it shows that the three exchange rate return p-values are all greater than 5%. Therefore we fail to reject the null hypothesis for the three series. It means we accept the null hypothesis that the residuals are random for all the return series, and they are independent and identically-distributed.

Finally, ARCH-LM Diagnostic Test was also used to check whether their is ARCH effect. Testing the null hypothesis of no ARCH effect against the alternative of ARCH effect. The Lagrange-Multiplier test in the table 2.1 offer significant evidence of ARCH effect, an indication that the series are candidates of GARCH type modeling.

Histogram density plot is also use to check for normality. According to the histogram density graphs above, all the three exchange rate returns looked not normally distributed. It shows the absence of normality for the three situations. The evolution plots show that all the three return series have volatility clustering, i.e.

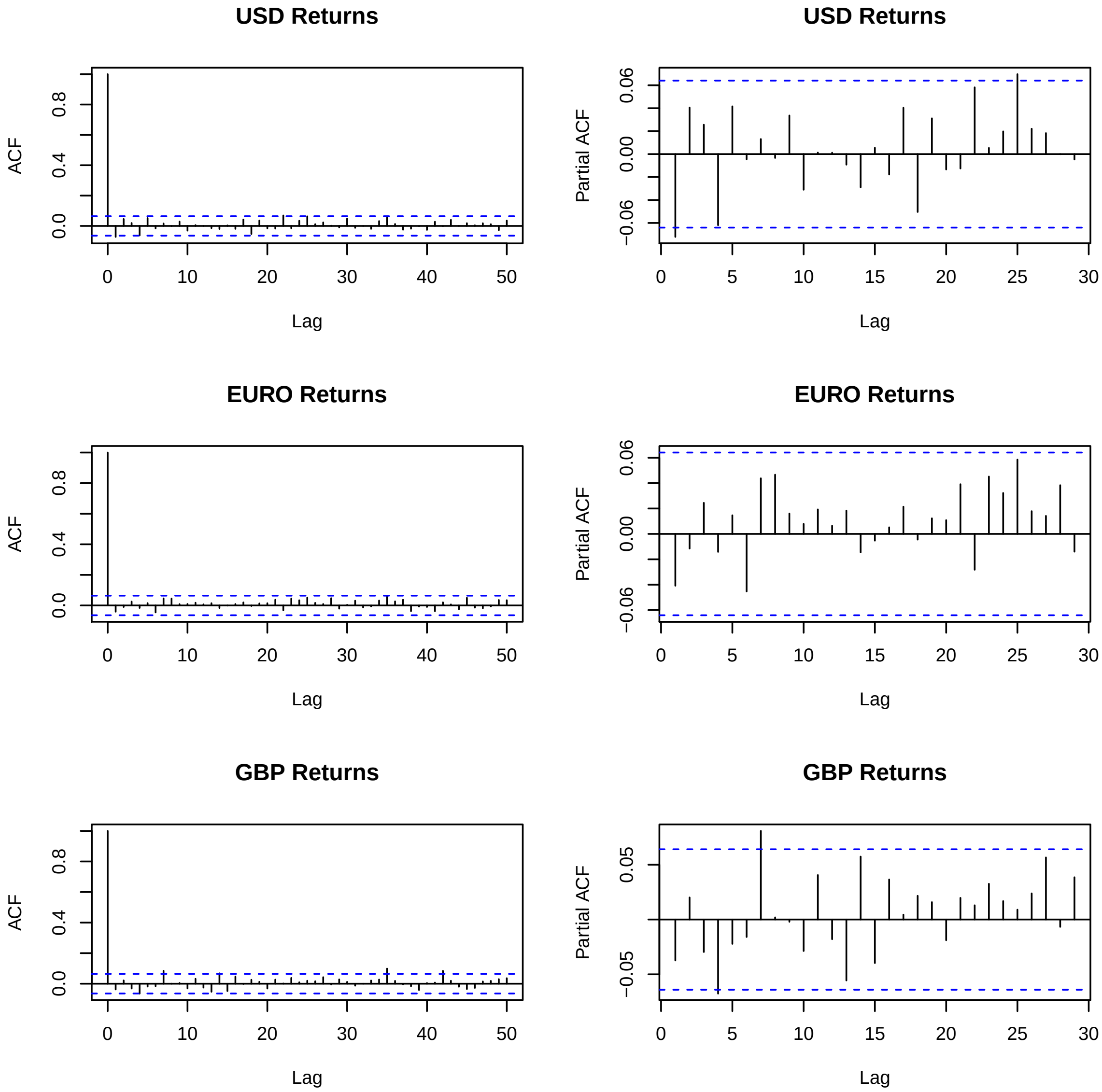
Fig. 1. Evolution and Density plot of the three return series



periods with high volatility and periods with low volatility, which indicates that a GARCH model can be used to fit the data.

The ACF plots for the three exchange rate returns seem to be white noise, all the lags fall inside the confidence interval. It also indicates no correlation and quickly decaying to zero. The PACF plots for the EURO shows that, all the lags are inside

Fig. 2. ACF and PACF plot of the three return series



the confidence bound and at most two lags are outside the confidence bound for the USD and GBP, thus there is evidence that the squared returns is predictable.

3. Methodology

3.1. The GARCH Models and its extensions

In this paper, we use both univariate and multivariate models. The univariate GARCH models are more interested in the sensitivity and persistence of a variable volatility shock on itself. Conversely, multivariate GARCH models (MV-GARCH) analyze the impact of the volatility of a variable on another variable. The most obvious application of MV-GARCH models is the study of relationships between co-volatilities and volatility in several markets.

3.1.1. Univariate Model

Understanding and analyzing how the univariate GARCH model works is the base for a study of the DCC multivariate GARCH model of Engle and Sheppard (2001). The GARCH(p,q) model introduced by Bollerslev (1986) is defined as:

$$r_t = \mu_t + \epsilon_t \tag{1}$$

$$\epsilon_t = h_t z_t \tag{2}$$

$$h_t^2 = \alpha_0 + \alpha_1 \epsilon_{t-1}^2 + ... + \alpha_q \epsilon_{t-q}^2 + \beta_1 h_{t-1}^2 + \cdots + \beta_p h_{t-p}^2 \tag{3}$$

Where r_t is the log return of an asset at time t, ϵ_t is the mean-corrected return of an asset at time t, μ_t is the expected value of the conditional r_t, h_t is the conditional variance at time t conditioned on the past, z_t is the sequence of independent and identically distributed (iid) standardized random variables thus $E[z_t] = 0$ and $Var[z_t] = 1$, $\alpha's$ and the $\beta's$ are the parameters of the model and (p, q) is the order of the GARCH model. The coefficients α_i $(i = 1, 2, ..., q)$ and β_j $(j = 1, 2, ..., p)$ are all assume to be positive to ensure that the conditional variance h_t^2 is always positive and $\sum_{i=1}^{q} \alpha_i + \sum_{j=1}^{p} \beta_j < 1$ ensures that the process is stationary.

In the literature, the most considered GARCH model is the case where $p = q = 1$. Notice that, the GARCH(p, q) does not include the asymmetry of the errors, which is a drawback. Some of the extended GARCH models accommodate the asymmetry of the returns such as the EGARCH model by Nelson (1991) and the GJR-GARCH model of Glosten et al. (1993).

3.1.2. EGARCH model

$$\ln h_t^2 = \alpha_0 + \sum_{i=1}^{q} \alpha_i \epsilon_{t-i}^2 + \sum_{j=1}^{p} \beta_j \ln h_{t-j}^2$$

$\epsilon_{t-i}^2 h_{t-i}^2$ are not i.i.d. and the conditional variance is invariant to changes in sign of the ϵ_t's.

When p=q=1, we have

$$\ln h_t^2 = \alpha_0 + \alpha\epsilon_{t-1}^2 + \beta \ln h_{t-1}^2. \tag{4}$$

3.1.3. GJR-GARCH model

The GJR-GARCH model was named after the authors who introduced it, Glosten et al. (1993). It extends the standard GARCH (p, q) to include asymmetric terms that capture an important phenomenon in the conditional variance.

$$h_t^2 = \alpha_0 + \sum_{i=1}^{q} \alpha_i \epsilon_{t-i}^2 + \sum_{i=1}^{q} \epsilon_{t-i}^2 \gamma_i I_{t-i} + \sum_{j=1}^{p} \beta_j h_{t-j}^2$$

where

$$I_{t-i} = \begin{cases} 1 & if \;\; \epsilon_{t-i} < 0 \\ 0 & if \;\; \epsilon_{t-i} \geq 0 \end{cases}$$

The conditional variance for a $gjrGARCH(1, 1)$ is

$$h_t^2 \;=\; \alpha_0 + \alpha\epsilon_{t-1}^2 + \epsilon_{t-1}^2 \gamma I_{t-1} + \beta h_{t-1}^2$$

In sample, to select the best GARCH model, the AIC and BIC model selection information criteria are considered.

3.1.4. Multivariate DCC GARCH model

The multivariate GARCH model proposed in this paper is the Dynamic Conditional Correlation (DCC) model introduced by Engle and Sheppard (2001) which offers a simple and more parsimonious means of modeling multivariate volatility.The DCC-GARCH belongs to the class models of conditional variances and correlations. The idea of the models in this class is that the covariance matrix H_t can be decomposed into conditional standard deviations D_t and a correlation matrix R_t. In the DCC-GARCH model both D_t and R_t are designed to be time varying. A DCC model can be defined as:

Let r_t be a vector of $(n \times 1)$ of a stationary process, r_t ∼DDC-GARCH is:

$$r_t = \mu_t + \epsilon_t \tag{5}$$

$$\epsilon_t = H_t^{\frac{1}{2}} z_t \tag{6}$$

$$H_t = D_t R_t D_t \tag{7}$$

Where D_t is the diagonal of an $n \times n$ matrix of time varying standard deviation from univariate GARCH models and R_t is the time varying conditional correlation matrix of the disturbance term ϵ_t.
D_t and R_t are in the forms:

$$D_t = \begin{bmatrix} \sqrt{h_{it}} & 0 & 0 & \cdots & 0 \\ 0 & \sqrt{h_{it}} & 0 & \cdots & 0 \\ 0 & 0 & \sqrt{h_{it}} & & \vdots \\ \vdots & \vdots & & \ddots & 0 \\ 0 & 0 & \cdots & 0 & \sqrt{h_{it}} \end{bmatrix} \quad R_t = \begin{bmatrix} 1 & q_{12,t} & q_{13,t} & \cdots & q_{1n,t} \\ q_{21,t} & 1 & q_{23,t} & \cdots & q_{2n,t} \\ q_{31,t} & q_{32,t} & 1 & & \cdots \\ \vdots & \vdots & & \cdots & \ddots \\ q_{n1,t} & q_{n2,t} & q_{n3,t} & \cdots & 1 \end{bmatrix}$$

where n is the sample size.

3.2. Estimation of DCC-GARCH

Here we describe how the parameters of a DCC-GARCH model may be determined. We consider two different distributions for the standardized error z_t; the multivariate Gaussian distribution and the multivariate Student's t-distribution.

3.2.1. Multivariate Gaussian distributed errors

When the standardized errors, z_t, are multivariate Gaussian distributed, the joint distribution of $z_1, ..., z_T$ is:

$$f(z_t) = \prod_{t=1}^{T} \frac{1}{(2\pi)^{\frac{n}{2}}} exp - \frac{1}{2} z_t^T z_t$$

The model estimation is done in two steps: The first step is to estimate the conditional variance of the series with a univariate GARCH $\log_v(\theta)$ and the second is to use the standardized residuals obtained to estimate the parameters of the dynamic correlation matrix $\log_c(\theta, \phi)$.
The log Likelihood function in the first step is:

$$\log(L(\phi)) = -\frac{1}{2} \sum_{t=1}^{T} ([\log h_t + \frac{\epsilon_t^2}{h_t}] + constant)$$

The log likelihood in the second step is:

$$\log(L(\psi)) = -\frac{1}{2} \sum_{t=1}^{T} (n \log(2\pi) + 2 \log(|D_t|) + \log(|R_t|) + \epsilon_t' R_t^{-1} \epsilon_t)$$

$\psi = (\mathrm{a}, \mathrm{b})$ are the parameters of the correlation. Since D_t is constant when conditioning on the parameters from step one, we can exclude the constant terms and maximize:

$$\log(L(\psi)) = -\frac{1}{2}\sum_{t=1}^{T}(n\log(2\pi) + \log(|R_t|) + \epsilon_t' R_t^{-1}\epsilon_t)$$

3.2.2. Multivariate Student's t-distributed errors

When the standardized errors, z_t, are multivariate Student's distributed, the joint distribution of $z_1, ..., z_T$ is:

$$f(z_t|\nu) = \prod_{t=1}^{T}\frac{\Gamma(\frac{\nu+n}{2})}{\Gamma(\frac{1}{2}[\pi(\nu-2)]^{\frac{n}{2}})}[1+\frac{z_t^T z_t}{\nu-2}]^{-}\frac{n+\nu}{2}$$

where $\Gamma(.)$ is the gamma function.

$$\Gamma(\alpha) = \int_0^\infty x^{\alpha-1}e^{-x}dx \quad \text{for } \alpha > 0.$$

Engle and Sheppard (2001) have shown that the change of the error distribution does not virtually affect the parameters. Therefore the parameters of the univariate GARCH models are fitted using the pseudo-maximum-likelihood. Assuming the errors to be Gaussian distributed, the log Likelihood function in the first step is the same as the one in the multivariate Gaussian.

The log likelihood in the second step is:

$$\begin{aligned}\log(L(\psi)) &= \sum_{t=1}^{T}(log[\Gamma(\frac{\nu+n}{2})] - log[\Gamma(\frac{1}{2})] - \frac{n}{2}log[\pi(\nu-2)] - \frac{1}{2}log[|R_t|] - log[|D_t|] \\ &- \frac{\nu+2}{2}log[1+\frac{\epsilon_t^T R^-1_t\epsilon_t}{\nu-2}])\end{aligned}$$

Since D_t is constant when conditioning on the parameters from step one, we can exclude the constant term and maximize:

$$\log(L(\psi)) = \sum_{t=1}^{T}(log[\Gamma(\frac{\nu+n}{2})] - log[\Gamma(\frac{1}{2})] - \frac{n}{2}log[\pi(\nu-2)] - \frac{1}{2}log[|R_t|] - \frac{\nu+2}{2}log[1+\frac{\epsilon_t^T R_t^{-1}\epsilon_t}{\nu-2}])$$

The proposed dynamic correlation structure For a $\mathrm{DCC(p,q)}$

$$
\begin{aligned}
Q_t &= (1 - \sum_{i=1}^{P} \alpha_i - \sum_{j=1}^{Q} \beta_j)\bar{Q} + \sum_{i=1}^{P} \alpha_i(\epsilon_{t-i}\epsilon'_{t-i}) + \sum_{j=1}^{Q} (\beta_j) Q_{t-1} \\
R_t &= Q_t^{*-1} Q_t Q_t^{*-1}
\end{aligned}
$$

$\bar{Q}$ is the unconditional covariance of the standardized residuals where Q_t is a positive definite matrix defining the structure of the dynamics and Q_t^{*-1} is a diagonal matrix with the square root of the diagonal elements of Q_t at the diagonal .

For a $\mathrm{DCC}(1,1)$

$$
\begin{aligned}
Q_t &= (1 - \alpha - \beta)\bar{Q} + \alpha\epsilon_{t-1}\epsilon'_{t-1} + \beta Q_{t-1}. \\
\bar{Q} &= cov(\epsilon_t\epsilon'_t) = E[\epsilon_t\epsilon'_t]
\end{aligned}
$$

where α and β are scalars.

Remark: The matrix R_t is positive if and only if the matrix Q_t is positive. So that H_t is positive, it is necessary that the following conditions are satisfied:

$$\alpha > 0;\ \beta > 0;\ \text{with } \alpha + \beta < 1.$$

Consider a DCC-GARCH$(1,1)$ multivariate model used in this study:

$$
\begin{aligned}
H_{11,t} &= \alpha_{0,1} + \alpha_{11}\epsilon^2_{1,t-1} + \beta_{11} H_{11,t-1} \\
H_{22,t} &= \alpha_{0,2} + \alpha_{22}\epsilon^2_{2,t-1} + \beta_{22} H_{22,t-1} \\
H_{33,t} &= \alpha_{0,3} + \alpha_{33}\epsilon^2_{3,t-1} + \beta_{33} H_{33,t-1}
\end{aligned}
$$

The parameters to be estimated are $\alpha_{0,1}$, $\alpha_{0,2}$ and $\alpha_{0,3}$ representing the average conditional volatility of series 1, 2 and 3; α_{11}, α_{22} and α_{33} are ARCH parameters measuring sensitivity and β_{11}, β_{22} and β_{33} are GARCH parameters that measure the persistence. Thus α_{ij} $\forall i, j = 1, 2, 3$ measures the sensitivity of the shock of the asset i on the asset j and β_{ij} $\forall i, j = 1, 2, 3$ measures the persistence of the shock of asset i on the asset j.

3.3. Forecasting of DCC-GARCH

After the parameters of the model are estimated, the next step is to determine the forecast of the conditional covariance matrix, $H_{t+k} = D_{t+k}R_{t+k}D_{t+k}$, at time $t + k$ when the history up to time t is known. When forecasting the covariance matrix, the forecasts of D_{t+k} and R_{t+k} can be separately done.

3.3.1. Forecasting the conditional variances in D_{t+k}

The forecasts of the univariate variances in $D_{t+k} = diag(\sqrt{h_{1,t+k}}, ..., \sqrt{h_{n,t+k}})$ can be done separately for each of the n assets. The k-step ahead forecast for the general $GARCH(q,p)$ is:

$$E[h_{i,t+k}|F_t] = \alpha_0 + \sum_{j=1}^{max(p,q)} (\alpha_j + \beta_j)E[h_{i,t+k-j}|F_t]$$

where

$$F_t = h_{i,1}, h_{i,2}, ..., h_{i,n}$$

and

$$E[h_{i,t+k}|F_t] = z^2_{i,t+k},\ for\ k < 0, i = 1, 2, \cdots, n.$$

The forecast of the conditional variance is

$$E[D_{t+k}|F_t] = diag(\sqrt{E[h_{1,t+k}|F_t]}, ..., \sqrt{E[h_{1,t+k}|F_t]}).$$

3.3.2. Forecasting the conditional correlation matrix R_{t+k}

The elements of R_{t+k} are not themselves forecast but they are the ratio of the forecast of the conditional covariance to the square root of the product of the forecasts of the conditional variances.

The expectation of the positive defined matrix Q_{t+k} is:

$$\begin{cases} E[Q_{t+1}|F_t] = (1-a-b)\bar{Q} + a\epsilon_t\epsilon_t^T + bQ_t\ \ if\ \ k = 1 \\ E[Q_{t+k}|F_t] = (1-a-b)\bar{Q} + aE[\epsilon_{t+k-1}\epsilon_{t+k-1}^T|F_t]b[Q_{t+k-1}|F_t]\ \ if\ \ k > 1 \end{cases}$$

where $E[\epsilon_{t+k-1}\epsilon_t^T|F_t] = E[R_{t+k-1}|F_t] = E[Q^{*-1}_{t+k-1}Q_{t+k-1}Q^{*-1}_{t+k-1}|F_t]$. We cannot directly compute the $k-step$ ahead forecast because $E[Q^{*-1}_{t+k-1}Q_{t+k-1}Q^{*-1}_{t+k-1}|F_t]$ is unknown. To approximate the forecast, we assumed $\bar{R} \approx \bar{Q}$ and $E[R_{t+i}|F_t] \approx E[Q_{t+i}|F_t]$ for $i = 1, ..., k$

$$\begin{aligned}
E[R_{t+i}|F_t] &\approx E[Q_{t+i}|F_t] \\
&\approx (1-a-b)\bar{Q} + aE[R_{t+k-1}|F_t] + bE[Q_{t+k-1}|F_t] \\
&\approx (1-a-b)\bar{R} + (a+b)E[R_{t+k-1}|F_t] \\
&\approx (1-a-b)\bar{R} + (a+b)(1-a-b)\bar{R} + (a+b)E[R_{t+k-2}|F_t] \\
&\approx (1-a-b)\bar{R} + (1-a-b)\bar{R}(a+b) + (a+b)E[R_{t+k-2}|F_t] \\
&\approx \cdots \\
&\approx \sum_{i=0}^{k-2}(1-a-b)\bar{R}(a+b)^i + (a+b)^{k-1}E[R_{t+1}|F_t] \\
&\approx (1-(a+b)^{k-1})\bar{R} + (a+b)^{k-1}E[R_{t+1}|F_t], \qquad (8)
\end{aligned}$$

where $E[R_{t+1}|F_t] \approx \hat{Q}^{*-1}_{t+1}\hat{Q}_{t+1}\hat{Q}^{*-1}_{t+1}, \hat{Q}_{t+1} = (1-a-b)\bar{Q} + a\epsilon_t\epsilon_t^T + bQ_t$ and $\bar{R} = \bar{Q}^*\bar{Q}\bar{Q}^*$, where $\bar{Q}^*$ is a diagonal matrix with the square root of the diagonal elements of Q on the diagonal.
There exist another method to approximate the forecast. See Engle and Sheppard (2001) for that method.

4. Applications

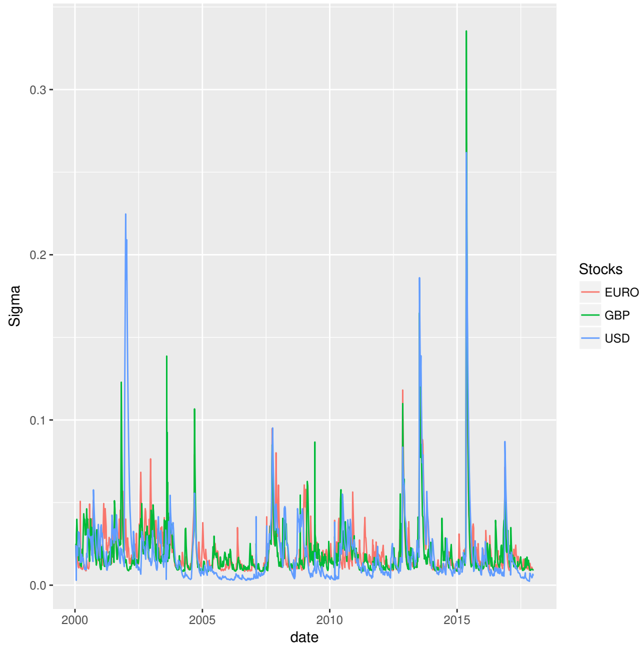
In this chapter, we use the Gambia exchange rate data for empirical Analysis. As it was shown in the data description part when the log returns were examined for heteroskedasticity, ARCH-LM test provides strong evidence of ARCH effects in the return series, which indicates that we can now proceed with the modeling of the index return volatility by using GARCH methodology (see Figure 1). The graph of the three series shows that, the USD has the highest volatility except around 2015/2016 were the GBP was the highest.

In the following, we check the GARCH model and some of the extension GARCH models. The different results are presented in Table 2.

Table 2. Comparison of Models using AIC and BIC for Gaussian and student distributed errors. Akaike's information criterion (AIC) and Bayes information criterion (BIC) compare the quality of a set of statistical models to each other.

	Gaussian			**Student**		
	GARCH	eGARCH	gjrGARCH	GARCH	eGARCH	gjrGARCH
Log-Likelihood	-4667.072	-4613.407	-4549.435	-4067.936	-4117.056	-4110.357
AIC	10.665	10.358	12.879	9.2646	9.2603	9.2645
BIC	10.789	10.499	13.020	9.4107	9.4027	9.4269

Fig. 3. Evolution plot of the three return series



The results in Table 3 considering the minimum Akaike Information Criteria (AIC) and Bayes information criterion (BIC) which estimate the quality of each model, relative to each of the other models, we can conclude that the eGARCH better fit the data for both the Gaussian and Student distributed error. Thus we consider eGARCH for the two distributed errors.

4.1. Parameter Estimation

First we fit ϵ_t assuming Gaussian distributed errors, z_t. When z_t $N(0, I)$, and ϵ_t $N(0, H_t)$. The estimated parameters from step one are given in Table 4.1.

Table 3 shows the results from estimating a trivariate eGARCH model using the weekly exchange rate returns, USD, EURO and GBP over the sample period 7th January, 2000 to 29th December, 2017, a total of 936 observation for each series. The parameters of the variance equation(α_1 and β_1) are all significant for the three series. The significance of α and β indicates that lagged conditional variance and squared disturbance has an impact on the conditional variance. In other words this means that news about volatility from the previous periods has an explanatory power on current volatility (H_t rely on H_{t-1}).

Table 3. Parameters from step 1 when assuming Gaussian distributed errors for EGARCH. Notes: ** is 5% significant. μ is the intercept of the conditional mean model, ω is the intercept of the GARCH(1,1) model, α_1 is the ARCH term, β_1 is the GARCH term and γ_1 is the asymmetric effect term.

	Estimate	Std. Error
USD		
μ	0.007953	0.025957
ω	0.120973**	0.046317
α_1	0.115062**	0.067615
β_1	0.870029**	0.013344
γ_1	0.539885**	0.085296
EURO		
μ	0.048751	0.054049
ω	0.270102**	0.084591
α_1	0.255397**	0.108687
β_1	0.834349**	0.052409
γ_1	0.731454**	0.139884
GBP		
μ	0.021257	0.050759
ω	0.367869**	0.138880
α_1	0.261654**	0.158252
β_1	0.720866**	0.051458
γ_1	0.979685**	0.231517

Since we, as described iabove, assumed Gaussian distributed errors in step one i.e univariate model, the estimated parameters in this step are exactly the same as the parameters given in Table 3.

Gaussian	Estimate	Std. Error
dcca1	0.057540**	0.023076
dccb1	0.641737**	0.156908
Student	Estimate	Std. Error
dcca1	0.086182**	0.025312
dccb1	0.759199 **	0.057061
mshape	4.000001**	0.622391

Table 4. Parameters from step 2 when assuming Gaussian and student distributed errors. *NB. dcca1* and *dccb1* are the ARCH and GARCH parameters of the estimated DCC model and mshape is the estimated parameter of the DCC for Student distribution

The estimated parameters in step two, summarized in Table 4.1, are also significant i.e conditional correlation parameters dcca1 and dccb1 are statistically significant for both distributed errors. The parameter mshape is also significant which corresponds to the degree of freedom for the student distributed error.

Test	Gaussian T-statistic	p-value	Student T-statistic	p-value
$Q(m)$ of ϵ_t:	2.861338	0.984	5.660861	0.842
Rank-based test:	64.36503	5e-10	60.1749	3e-09
$Qk(m)$ of ϵ_t:	86.81395	0.575	136.1427	0.001
Robust $Qk(m)$:	142.5182	.0003	130.7848	0.003

Table 5. Residuals Test Results

As shown in Tsay (2014), Robust $Qk(m)$) works well in detecting conditional heteroscedasticity of multivariate return series. According to the join Robust statistic test, as interpreted in Tsay (2014), of the three return series, presented in Table 4.1, shown that the p-values for both the Gaussian and Student distributed errors are statistically significant at the 5% confidence level with 90 degrees of freedom of a chi-squared distribution. This confirms the existence of serial dependence in the three return series. Meaning that the three return series are correlated. Thus the two models are adequate for further analysis.

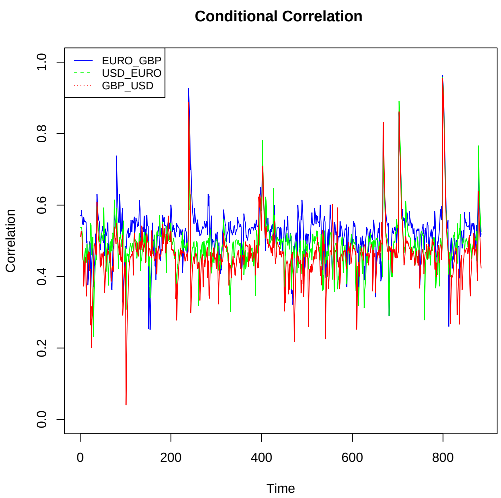


Fig. 4. Gaussian distributed error

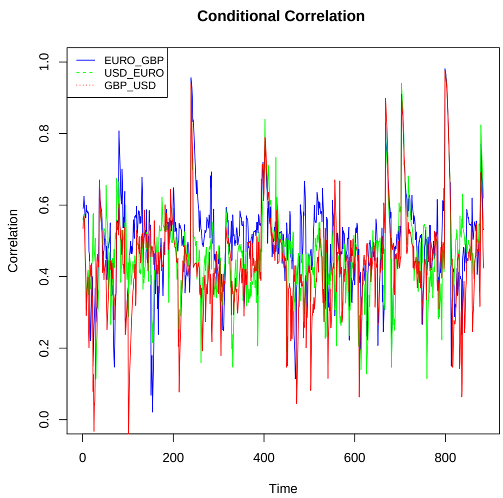


Fig. 5. Student distributed error

Figure 4 and 5 show the fitted, volatilities and correlations resulting from the Gaussian and Student eGARCH models respectively. According to the graphs, we see that the correlation between USD and GBP is more volatile in both cases. The student distributed error shows negative correlation around 2002. The graphs, also show relatively stable and persistent conditional correlation. They also show that the moving pattern are centralized around an upward rising trend.

4.2. Forecasts

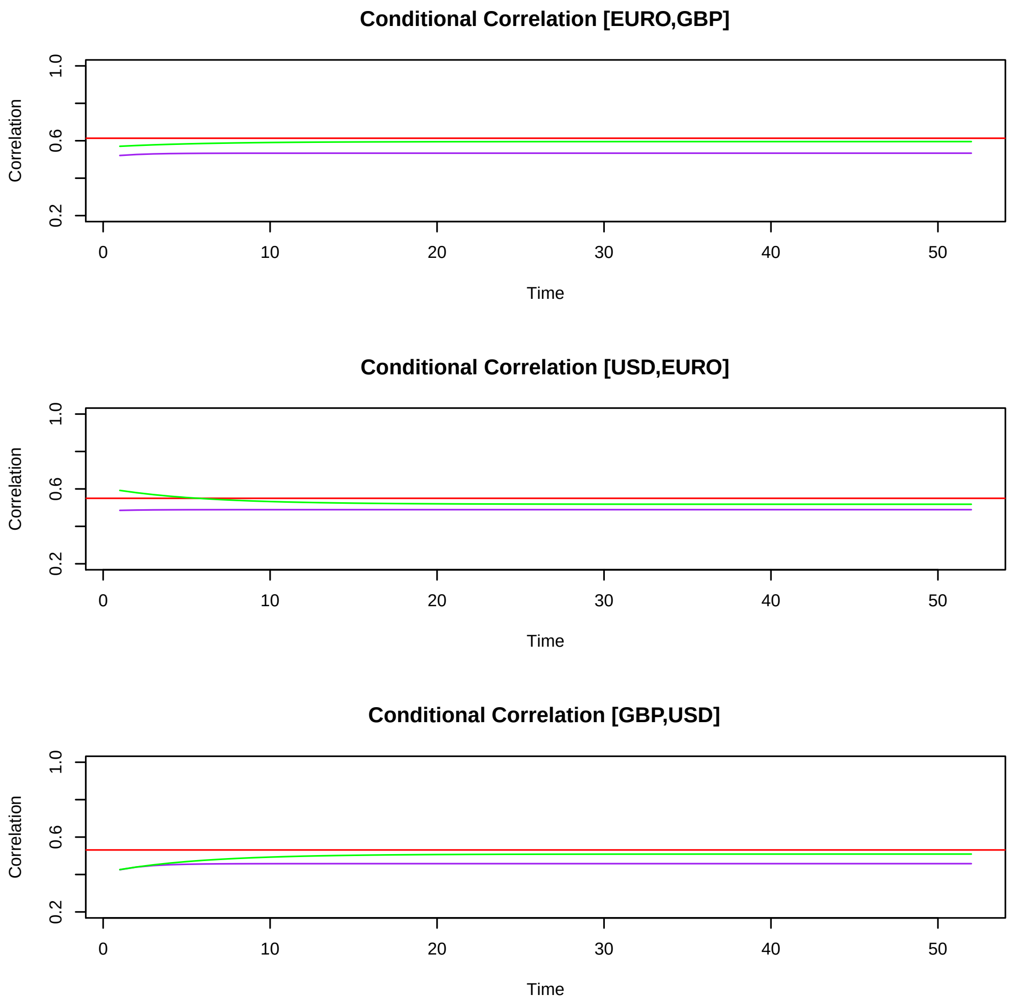
To complete the study, we forecast the distributed errors using the conditional correlation matrix. Figure 6 shows the forecast off diagonals for the correlation matrix of the Gaussian and student distributed errors. The red line is the unconditional correlation matrix, the purple line is the one year forecast for the Gaussian distributed error and the green line is the one year forecast of the student distributed error. In all the correlation forecast, we see that the green line i.e student distributed error is closer to the unconditional correlation. This shows that the student distributed error is better. The graph also indicates constant correlation except around the first week to the eight week of the USD and EURO correlation for Student distributed error which was a bit above the unconditional correlation. The Student and Gaussian distributed errors show the same correlation between GBP and EURO around the first week to the fifth week. The student distributed error for EURO and GBP is closer to the unconditional correlation than the correlation between the USD and EURO and the correlation between GBP and USD. This shows that the EURO and GBP has the highest correlation.

5. Conclusion

In this study we investigated the statistical properties of The Gambia exchange rates (USD, EURO and GBP) from 2000 to 2017 and specify two distributed errors namely the Gaussian and the Student distributed errors. The data was divided into two (in-sample and out-sample) and fit the DCC eGARCH(1,1) model for both distributed errors. The models for the two distributed errors were selected using some of the in-sample characteristic AIC and BIC and estimated the parameter which were significant.The sum of the GARCH parameters α_1 and β_1 were found to be very close to one, indicating that volatility in the exchange rate is highly persistent. The remaining part of the data set was used for out-of-sample forecasting of the conditional correlation. Our results indicate that the exchange rates of The Gambia Dalasi are correlated and the student distributed errors better forecast the conditional correlation.

The variation of weekly exchange rates of USD, EURO and GBP currencies against the Gambian Dalasi shows instability behavior during the selected period which is associated with exchange rate risk. This fluctuation is considered non-stationary and has been frequently observed in a number of time-series data applied in previous studies. The movement shows an upward trend, indicating that the currency Dalasi against the three international currencies has been depreciating over the past decades. One of the main factors of this nature is the decline of the country's re-export trade due to the timing of external tariffs in the developments of the state to compete and effective port facilities, mainly in Senegal.

Fig. 6. Forecast for the Gaussian and student distributed errors



This study will help policy directives for those working in the central Bank of The Gambia, Government Ministries and Financial institutions to fulfill some of the pronouncements in The Gambian constitution and Act (2005) where one of the main objectives is to maintain and stabilize the prices of goods and currencies. Being one of the import led nations and depending a lot on remittances coming from abroad, there is the need to conduct research and studies of trends and volatility of the exchange rates of The Gambia over a period of time, to achieve some of these goals.It may be of interest to future researchers to use Value-at-Risk to measure the exchange rate risk and to get a better analysis of how volatility affects the exchange rate of The Gambian Economy and not just the market, it might be better to use other distribution errors such as a skewed Student and Normal Inverse Gaussian (NIG). In our analysis, for both the Student and Gaussian distribution, we assumed the same distribution for the three series. This might be very restrictive Criteria. One then might use another multivariate GARCH model (Copula-GARCH) to link the marginals together for different distribution assumption of the three series which might better fit the data.

Acknowledgment The authors would like to greatly thank the editor and the presentation. Special thanks are extended to the referees for the careful reading and valuable comments which greatly helped in improving the paper.

References

Abdalla, S. and Zakaria, S.(2012). Modeling Exchange Rate Volatility using GARCH Models: Empirical Evidence from Arab Countries. *International Journal of Economics and Finance*, Vol. 4 (3), pp. 216-229.

Arranz, M. and Iglesias, E. (2005). An àlisis de los tipos de cambio en la economá mexicana y comparación con otros países: un enfoque de volatilidad estocística. *Investigación económica*, Vol. 64 (253), pp. 159-169.

Baillie, R. T., Bollerslev, T. and Mikkelsen, H. O. (1996). Fractionally integrated generalized autoregressive conditional heteroskedasticity. *Journal of econometrics*, Vol. 74 (1), pp. 3-30.

Bauwens, L. and Sucarrat, G. (2010). General-to-specific modeling of exchange rate volatility: A forecast evaluation. *International Journal of Forecasting*, Vol. 26 (4), pp. 885-907.

Bollerslev, T. (1986). Generalized Autoregressive Conditional Heteroskedasticity. *Journal of Econometrics*, Vol. 31 (3), pp.307-327.

Bollerslev, T. (1990). Modelling the Coherence in Short-run Nominal Exchange Rates: A Multivariate Generalized ARCH Model. *The Review of Economics and Statistics, MIT Press*, Vol. 72 (3), pp. 498-505, August

Bosnjak, M., Novak, I. and Bilas, V. (2016). Modeling exchange rate volatilities in Croatia. *Ekonomski vjesnik/Econviews Review of Contemporary Business, Entrepreneurship and Economic*, Vol. 29 (1), pp. 81-93.

Orskaug, E. (2009). Multivariate DCC-GARCH Model. *Master Thesis of Norwegian University of Science and Technology*.

Engle, R. F. (1982). Autoregressive Conditional Heteroskedasticity with Estimates of the Variance of U.K. Inflation. *Econometrica*, Vol. 50 (4), pp. 987-1008.

Engle, R.F. (2002). Dynamic Conditional Correlation: A Simple Class of Multivariate GARCH Models. *Journal of Business and Economic Statistics*, Vol. 20 (3), 339-350.

Engle, R. F. and Sheppard, K. (2001). Theoretical and Empirical Properties of Dynamic Conditional Correlation MV-GARCH. *NBER Working Paper No. 8554*.

Glosten, L.R., Jagannathan, R. and Runkle, D.E. (1993). Relationship between the expected value and the volatility of the nominal excess return on stocks. *The Journal of Finance*, Vol. 48 (5), pp. 1779-1801

Hartman, J. and Sedlak, J. (2013). Forecasting Conditional Correlation for Exchange Rates using Multivariate GARCH models with Historical Value-at-Risk application. *Bachelor Thesis Uppsala University*.

Longmore, R. and Robinson, W. (2004). Modelling and forecasting exchange rate dynamics: an application of asymmetric volatility models. *Bank of Jamaica Working Paper WP2004/03*.

Maana, I., Mwita, P. N. and Odhiambo, R. (2010). Modeling the volatility of exchange rates in the Kenyan market. *African Journal of Business Management*, Vol. 4 (7), pp. 1401-1408.

MacDonald, R. and Taylor, M. P. (1994). The monetary model of the exchange rate: long-run relationships, short-run dynamics and how to beat a random walk. *Journal of international Money and finance*, Vol. 13 (3), pp. 276-290.

Mahieu, R. and Schotman, P. (1998). An empirical application of stochastic volatility models. *Journal of Applied Econometrics*, Vol. 13 (4), pp. 333-360.

McMillan, D. G., Speight, A. E. (2010). Return and volatility spillovers in three Euro exchange rates. *Journal of Economics and Business*, 62 (2), pp. 79?93.

Marreh, S., Olubusoye, O. E. and Kihoro, J. M. (2014). Modeling Volatility in the Gambian Exchange Rates: An ARMA-GARCH Approach. *International Journal of Economics and Finance*, Vol. 6 (10), pp. 118-128.

May, C. and Farrell, G. (2017). Modeling exchange rate volatility dynamics: Empirical evidence from South Africa. *Economic Research Southern Africa (ERSA), working paper 705*.

Nelson, D. B. (1991). Conditional Heteroskedasticity in Asset Returns: A new approach. *Econometrica*, Vol. 59 (2), pp. 347-370.

Olowe, R. A. (2009). Modeling naira/dollar exchange rate volatility: application of GARCH and asymmetric models. *International Review of Business Research Paper*, Vol. 5 (3), pp. 377-398.

Patton, A. (2004). On the Out-of-Sample Importance of Skewness and Asymmetric Dependence for Asset Allocation. *Journal of Financial Econometrics*, Vol. 2 (1), pp. 130-168.

Peters, T. (2008). Forecasting the covariance matrix with the DCC GARCH model. *Stockholm University, Examensarbete, 2008:4*

Sandoval, J. (2006). Do Asymmetric GARCH models fit better exchange rate volatilities on emerging markets? *ODEON*, Vol. 3, pp. 97-118.

Sarno, L., Taylor, M. P. and Chowdhury, I. (2004). Nonlinear dynamics in deviations from the law of one price: a broad-based empirical study. *Journal of International Money and Finance*, Vol. 23 (1), pp. 1-25.

Sengupta, J. K. and Sfeir, R. E. (1996). Exchange rate instability: some empirical tests of temporal dynamics. *Applied Economics Letters*, Vol. 4 (9), pp. 547-550.

Siddiqui, M. A. (2009). Modeling Pakistan rupee volatility against five major currencies in the perspective of different exchange rate regimes. *European Journal of Economics, Finance and Administrative Sciences*, Vol.17, pp. 81-96.

Taylor, S. J. (1986). Modeling Financial Time Series. World Scientific Publishing.

Tsay, R. S. (2014). Multivariate Time Series Analysis: with R and Financial Applications. Wiley.

Tse, Y. (1998). The conditional heteroscedasticity of the yen-dollar exchange rate. *Journal of Applied Econometrics*, Vol. 13 (1), pp. 49-55.