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A note on Computation of Multivariate Control Limits: The Bootstrap Approach

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Abstract. This work focuses on how to set control limits that will best identify signals in multivariate control charts. In any production process, every product is aimed to attain a certain standard, but the presence of assignable cause of variability affects our process thereby leading to low quality of product. However, the problem involved in the use of multivariate control chart is the violation of multivariate normal assumption. The first method develops bootstrap procedures to determine Hotelling's T^2 control limits for detecting large shift. The second method develops bootstrap procedures for obtaining Multivariate Exponentially-Weighted Moving Average (BMEWMA) control limits for identifying small shift. Results from a performance study shows that the proposed methods enable the setting of control limits that can enhance the detection of out of control signals.

Key words: run lengths, bootstrap methods, control limits, signals, multivariate distribution, quality.
AMS 2010 Mathematics Subject Classification Objects : 62F40; 62P30; 62G09.

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Résumé (French Abstract) Ce travail se concentre sur la détermination de limites dans un processus de contrôle statistique multivarié. Dans un processus de production, chaque élément produit doit répondre à des standards et des causes de variations données peuvent déranger le processus et conduire à des produits de qualité hors norme. Dans le cas précis du processus multivarié, les causes de dérèglements sont relatifs a la violation des hypothèses de normalité. D'abord, nous proposons une méthode de Boostrap pour obtenir des limites de contrôle de Hotelling. Ensuite, nous introdusons une méthode de Boostrap basée sur des moyennes mobiles multivariées de poids exponentiels. Des études de simulations montrent que ces méthodes sont aptes à détecter les produits hors-normes..

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1. Introduction

In Statistical Process Control (SPC), control charts play a vital role for the observance of a given process. For instance, the observance of process variables of manufactured products/items to ensure their adherence to specified standards (e.g. production processing of detergent soap, family delight pure soya oil, coca cola, cement, etc), monitoring of health data to determine an outbreak of a disease (case of ebola, laser fever, coronavirus etc), or the surveillance of a natural phenomenon such as flood control. Control charts are graphical representation for the purpose of identifying variations in a process under investigation. Signals occur in control chart when the statistic plotted falls either above or below the control limits. The limits of control charts are obtained using the distribution of the charting value and the desired false alarm rate. These limits are determined by statistical criteria in such a way that when the system is in control, most of the observations falls within the controls limits. In situations when the observations fall outside the control limits, the process is said to be out of control signals Mahmoud *et al.*(2010). Multivariate control charts are analogous to the univariate ones, they involve in the computations of several quality characteristics (d) instead of one Champ *et al.*(2005). For instance in Aparisi *et al.*(2004), if the population has a multivariate normal distribution having parameters with known mean vector (μ) and variance covariance matrix (Σ), the appropriate charting method to observe the mean is chi-square (χ^2) with its charting statistic as:

$$\chi^2 = (x-\mu)'\Sigma^{-1}(x-\mu) \tag{1}$$

The minimum control region (LCL) is zero and the maximum control region (UCL) for a given false alarm rate of α, is

$$UCL_{\chi^2} = \chi^2_{a,d} \tag{2}$$

where $\chi^2_{a,d}$ is obtained from the chi-square distribution having d degrees of freedom. When the parameters (μ and Σ) are not known and must be obtained from the given data assuming the process to be under control, the appropriate charting method to observe the mean is Hotelling's T^2 with its charting statistic as:

$$T^2 = (x - \bar{x})^{'} S^{-1}(x - \bar{x}) \tag{3}$$

and the control limits can be computed using the F-distribution. The minimum control region is zero and the maximum control region for a give rate of α is:

$$UCL_{T^2} = \frac{d(n+1)(n-1)}{n(n-d)} F_{\alpha,d,n-d} \tag{4}$$

where α is the specified probability of type 1 error rate (false alarm rate) and $F_{\alpha,d,n-d}$ represents the F distribution having parameters d and n-d degrees of freedom, n becoming the quantity of data and d the process observations. Multivariate Shewhart control charts are virtual not sensitive to detect little and modest shifts in process mean vector.

The Multivariate Exponentially-Weighted Moving Average (MEWMA) method is excellent in identifying small, moderate and large signals as a result of the scalar charting constant $\lambda(0 < \lambda \leqslant 1)$ which may be adjusted to change the weighting of the past observation Lowry *et al.*(1992). The chart uses the charting statistic

$$T_i^2 = Z_1^{'} \Sigma_{z_i}^{-1} Z_i, i = 1, 2, ... \tag{5}$$

where

$$Z_i = \lambda x_i + (1 - \lambda) z_{i-1} \tag{6}$$

and the covariance matrix is given by

$$\Sigma_{zi} = \frac{\lambda}{2 - \lambda} \left[1 - (1 - \lambda)^{2i}\right] \Sigma \tag{7}$$

with the scalar charting constant $\lambda(0 < \lambda \leq 1)$, Z_i is the vector of the set of data at time i where $z_0 = 0$. The MEWMA control chart often performs poorly when multivariate normal assumption is violated Sullivan *et al.*(2001). The charting constant may be chosen in a way that similar average run lengths are achieved under a wide range of distributions. The values acceptable for the charting constant are often very small, which means putting the majority of the weight on the past observations instead of the most current ones Stoumbos *et al.*(2002). However, problem involved in the use of multivariate control method is the violation of multivariate normal assumption that is required for many charts. The aims of this work are to: (a) initiate bootstrap methods in setting control limits as an option to the usual practices that observed data is from a normal distribution, and (b) to determine relatively the performance of Bootstrap T^2 and BMEWMA control limits as the bootstrap replications increase with the existing f-distribution method at selected α level of significance.

Performance evaluation of a control chart is known as average run length (ARL) and is the expected number of samples taken before the chart signals. During the in-control period, $ARL = \frac{1}{a}$ and is called ARL_0. The risk α is the well known as type I error (see **?**). Conventionally, the average run length (ARL) serves as a very useful and standard criterion for measuring the performance of a control chart scheme Lee *et al.*(2012). In-control ARL is related to the measure of the probability of type I error. The smaller the probability of type I error is, the longer the in-control ARL. In other words, a good control scheme should have long in-control ARL. When a process is out-of-control, there is a probability that the control scheme deems it as in-control; this is defined as the probability of type II error. A good monitoring scheme should have small probability of type II error. Out-of-control ARL is related to the measure of the probability of type II error. The smaller the probability of type II error is, the shorter the out-of-control ARL. And the shorter the out-of-control ARL is, the better the control scheme (see Lee *et al.*(2013)). In general, a good control scheme should have long in-control ARL and short out-of-control ARL. In this work, the multivariate bootstrap T^2 and BMEWMA control limits for monitoring out of control signals shall be computed and results compared with the f-distribution method. To measure the performance of a control chart, the Average Run Length (ARL), Standard Deviation Run Length (SDRL), Median Run Length (MRL) and Percentiles Run Length (PRL) shall be adopted.

2. Bootstrap Control Limits for Process Mean Vector and Variability

Besides the classical multivariate statistical process control (MSPC) technology, few scholars have been trying to monitor the abnormal situation in multivariate process via the bootstrap based multivariate control charts. The bootstrap technique was introduced by Polansky (2005) to determine univariate and multivariate control limits from numerical integration and discrete distribution. However, Alfaro *et al.*(2009), introduce bootstrap categorization method to discover multivariate out of control signals for quality variables. The bootstrap-based non-parametric control limits obtained via the cumulative sum charts was developed by Chatterjee *et al.*(2009). Phaladiganon *et al.*(2011) introduced the bootstrap method as a means of obtaining Hotellng's T^2 control limits assuming that the distribution is not multivariate normal by bootstrapping from Hotelling's T^2 statistic. For uncorrelated data, Adewara *et al.*(2012) obtained the minimum and maximum control limits by proposing the bootstrap method. Application of the minimax control chart by way of chi square control method for multivariate manufacturing process was also introduced by Balali *et al.*(2013). For estimating significant value and out of control limits for variables that are autocorrelated, the balance bootstrap method was proposed by Kalgonda (2013).

However, Gandy *et al.*(2013) introduced the methods of exact classified bootstrap control limits to monitor performance assessment. Control limits obtained from block bootstrap multivariate autocorrelated procedure based on Z-statistics was introduced by Kalgonda (2013). The method of setting the bootstrap multivariate exponentially weighted moving average control limits and the p-values for interpreting out of control signals was introduced by Ikpotokin *et al.*(2017). A comparative analysis of bootstrap multivariate exponentially-weighted moving average (BMEWMA) control limits was introduced by Ikpotokin *et al.*(2017). The bootstrap Bartlett adjustment on decomposed variance-covariance matrix of seemingly unrelated regression model was introduced by Alaba *et al.*(2019). To achieve the aims of this work, the bootstrap algorithm is given as follows:

ALGORITHM I: Bootstrap Method for obtaining Hotelling's T^2 Control Limit
Let there be d process characteristics such that every of the process characteristic includes n set of observations ($x_{ij}; i = 1, 2, ..., n; j = 1, 2, ..., d$), the proposed bootstrap procedure for obtaining Hotelling's T^2 control limits is as follows:

STEP 1. Merge the sample sizes of $x_1, x_2, ..., x_d$ of the sets of variables such that: $x = (x_{11}, x_{21}, ..., x_{n1}; x_{12}, x_{22}, ..., x_{n2}; ...; x_{1d}, x_{2d}, ..., x_{nd})$

STEP 2. Select a bootstrap sample of size $x^* = x_1^*, x_2^*, ..., x_d^*$ with replacement from Step (1) $x^* = x_{11}^*, x_{21}^*, ..., x_{n1}^*; x_{12}^*, x_{22}^*, ..., x_{n2}^*; ...; x_{1d}^*, x_{2d}^* ..., x_{nd}^*$

STEP 3. Replicate Step (2) for many numbers of periods as well as estimating bootstrap replications as: $x^* = x_{11}^{*(i)}, x_{21}^{*(i)}, ..., x_{n1}^{*(i)}; x_{12}^{*(i)}, x_{22}^{*(i)}, ..., x_{n2}^{*(i)}; ...; x_{1d}^{*(i)}, x_{2d}^{*(i)}, ..., x_{nd}^{*(i)}$, where $(i^* = 1, 2, ..., B)$.

STEP 4. Compute bootstrap mean vector $(\bar{x}^*)$, bootstrap variance and covariance matrix (S^*) from bootstrap samples in Step (3)

STEP 5. Find the bootstrap Hotelling's T_i^{2*} statistic from Step (4) such that: $T_i^{2*} = (x_j^* - \bar{x}*)' S^{*-1}(x_j^* - \bar{x}^*), i^* = 1, 2, ..., B; j^* = 1, 2, 3, ..., d.$

STEP 6. For bootstrap replications, repeat the processes in Step (5) B number of times by varying the values of T_i^{2*} and x_j^* to attain T_1^{2*},$T_2^{2*}, ..., T_B^{2*}$.

STEP 7. Obtain the control limit in such a way that each of the bootstrap statistic $(T_1^{2*}, T_2^{2*}, ..., T_B^{2*})$ is prearranged from the smallest to highest number, and establish the point of $B(1-\alpha)^{th}$ such that:

$$CL_{Boot(T^2)} = \#(T_1^{2*}, T_2^{2*}, ..., T_B^{2*}) \leq B(1-\alpha) \tag{8}$$

STEP 8. Determine the average run length (ARL), standard deviation run length (SDRL), median run length (MRL), and percentiles run lengths (PRL) from the repeated bootstrap T_i^{2*} statistic, $T_1^{2*}, T_2^{2*}, ..., T_B^{2*}$.

ALGORITHM II: Bootstrap Multivariate Exponentially Weighted Moving Average (BMEWMA) for Setting Control Limits.
Adopting Step 1 to Step 4 of Algorithm I, Steps 5 to 8 are introduced as following:

STEP 5. Determine BMEWMA (T_i^{2*}) statistics from Step (4) as

$$T_i^{2*} = Z_i^{*'} \Sigma_{Z_i^*}^{-1} Z_i^*, \tag{9}$$

where

$$Z_i^* = \lambda x_1^* + (1-\lambda) Z_{i-1}^* \tag{10}$$

$$\Sigma_{Z_i^*} = \frac{\lambda}{2-\lambda}\left[1-(1-\lambda)^{2i}\right]\Sigma^* \qquad 0 < \lambda \leq 1 \tag{11}$$

STEP 6. For bootstrap replications, repeat the processes in Step (5) B number of times by varying the values of Z_B and Σ_{Z_B} accordingly to attain T_1^{2*},T_2^{2*},...,T_B^{2*}.

STEP 7. Obtain the control limit in such a way that each of the bootstrap statistic (T_1^{2*},T_2^{2*},...,T_B^{2*}) is prearranged from the smallest to highest number, and establish the point of $B(1-\alpha)^{th}$ such that:

$$CL_{BMEWMA} = \#(T_1^{2*}, T_2^{2*}, ..., T_B^{2*}) \leqslant B(1-\alpha) \tag{12}$$

STEP 8 Determine the average run length (ARL), standard deviation run length (SDRL), median run length (MRL), and percentiles run lengths (PRL) from the repeated bootstrap T_i^{2*} statistic, $T_1^{2*}, T_2^{2*}, ..., T_B^{2*}$

3. Application to Numerical Example

Four quality characteristics denoting active detergent (X_1), moisture content (X_2), bulk density (X_3) and ph level (X_4) are set of data used in this work which is on detergent production processing obtained from Oyeyemi (2011). Implementing bootstrap algorithms I & II which were translated to visual basic code, bootstrap samples are generated 250, 500, 1000, 3000, 5000, 7500, 10000 occasions from the initial data set and their bootstrap control limits (BCL) are computed such that the false alarm rate fixed to values of α = 0.01, 0.025, 0.05, 0.1, 0.2, 0.25 as shown in Table 1. For instance, $\alpha = 0.05$ for B = 3000 is BCL=10.653 (i.e. the position of 2850 in Step 7 for T^2 sorted), and the f-distribution control limits (FCL) are computed by adopting Equation (4). While Table 2 show their various Run Lengths (RL) such as: Average Run Length (ARL), Standard Deviation Run Length (SDRL), Median Run Length (MRL) and Percentiles Run Length (PRL). Figures 1-7 is showing a control limits compared using the three methods when B = 250, 500, 1000, 3000, 5000, 75000, 10000, and n = 35 (sample size) fixed. However, Figures 8-13 is showing a comparison of ARL, SDRL, MRL and PRL computed for the Bootstrap Methods

Table 1. Control Limits for F-Distribution T^2, Bootstrap T^2 and BMEWMA at Selected α

Data Sample	**F-distribution T^2 Control Limits at α:** 0.05	0.01	0.025	0.05	0.1	0.2	0.25	0.5	0.75
35	20.713	18.017	14.592	12.087	9.636	7.204	6.416	3.871	2.167
Bootstrap Replication	**Bootstrap T^2 Control Limits** 0.05	**0.01**	**0.025**	**0.05**	**0.1**	**0.2**	**0.25**	**0.5**	**0.75**
250	13.155	12.469	11.484	9.744	7.534	5.745	5.291	3.406	1.777
500	17.465	14.46	11.609	9.596	7.922	5.904	5.393	3.442	2.014
1000	16.094	13.834	11.561	10.013	8.361	6.389	5.733	3.631	1.964
3000	18.04	15.992	12.605	10.653	8.763	6.833	6.099	3.711	2.079
5000	14.785	13.465	10.536	9.109	7.408	5.747	5.193	3.262	1.85
7500	15.915	14.057	12.129	10.205	8.368	6.474	5.617	3.517	2.046
10000	14.878	13.156	11.161	9.534	7.746	5.967	5.368	3.351	1.907
Bootstrap Replication	**BMEWMA Control Limits** 0.05	**0.01**	**0.025**	**0.05**	**0.1**	**0.2**	**0.25**	**0.5**	**0.75**
250	12.626	12.043	9.435	7.975	6.221	4.62	4.302	2.774	1.663
500	12.551	10.703	9.365	7.876	6.268	5.033	4.492	2.871	1.683
1000	12.06	10.546	8.999	7.545	6.413	4.982	4.472	2.831	1.569
3000	13.018	11.72	9.578	8.24	6.74	5.191	4.655	2.915	1.719
5000	13.037	11.788	9.832	8.213	6.829	5.174	4.672	2.9	1.686
7500	11.506	10.312	8.816	7.639	6.163	4.884	4.234	2.676	1.55
10000	11.363	10.18	8.679	7.433	6.081	4.644	4.18	2.599	1.506

Table 2. Run Lengths (RL) for Bootstrap T^2 and BMEWMA

Bootstrap Replication	Bootstrap T2 Run Lengths (RL) ARL	MRL	SDRL	25th Percentile	75th Percentile	95th Percentile
250	3.885	3.409	2.798	1.788	5.278	9.478
500	4.088	3.448	2.893	2.015	5.399	9.596
1000	4.238	3.633	2.988	1.969	5.733	10.013
3000	4.494	3.712	3.316	2.079	6.1	10.652
5000	3.863	3.262	2.739	1.853	5.193	9.109
7500	4.234	3.517	3.641	2.046	5.617	10.205
10000	3.985	3.351	2.825	1.907	5.368	9.534
Bootstrap Replication	**BMEWMA Run Lengths (RL) ARL**	**MRL**	**SDRL**	**25th Percentile**	**75th Percentile**	**95th Percentile**
250	3.31	2.7825	2.273	1.729	4.3	7.898
500	3.37	2.879	2.262	1.685	4.494	7.878
1000	3.309	2.836	2.317	1.569	4.472	7.547
3000	3.472	2.915	2.428	1.719	4.655	8.24
5000	3.493	2.9	2.463	1.686	4.672	8.213
7500	3.178	2.678	2.218	1.551	4.231	7.639
10000	3.111	2.599	2.198	1.506	4.18	7.433

Figures 1-7 Showing Results of Control Limits obtained when B = 250, 500, 1000, 3000, 5000, 75000, 10000, and n = 35

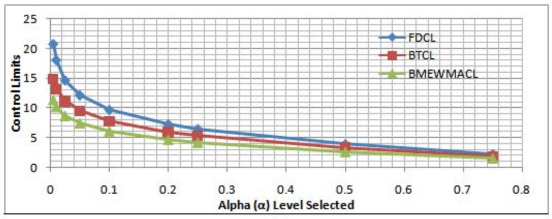

Figure 1: Control Limits for the three Methods Compared when B = 250 and n = 35

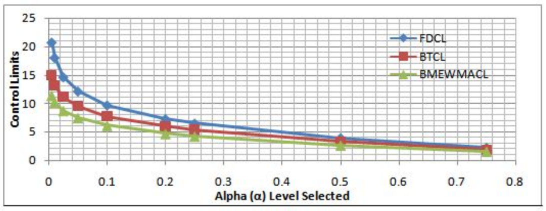

Figure 2: Control Limits for the three Methods Compared when B = 500 and n = 35

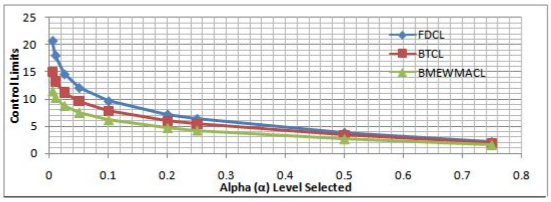

Figure 3: Control Limits for the three Methods Compared when B = 1000 and n = 35

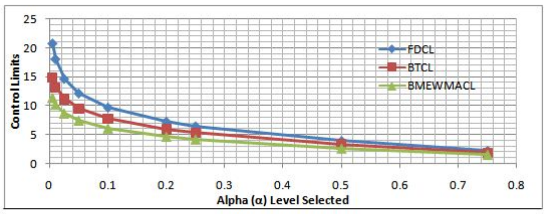

Figure 4: Control Limits for the three Methods Compared when B = 3000 and n = 35

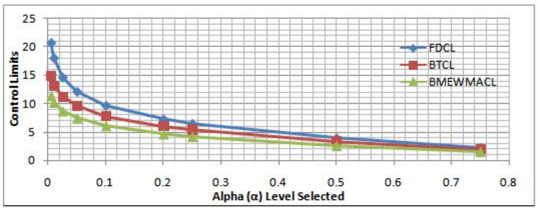

Figure 5: Control Limits for the three Methods Compared when B = 5000 and n = 35

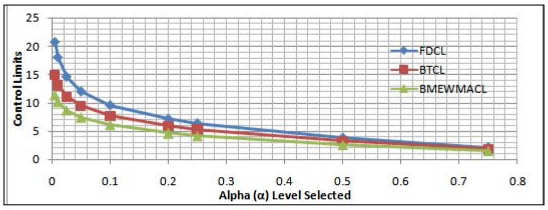

Figure 6: Control Limits for the three Methods Compared when B = 7500 and n = 35

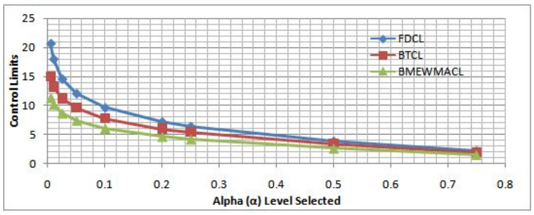

Figure 7: Control Limits for the three Methods Compared when B = 10000 and n = 35

Figures 8-13: Showing Results of ARL, SDRL, MRL and PRL computed for the Bootstrap Methods

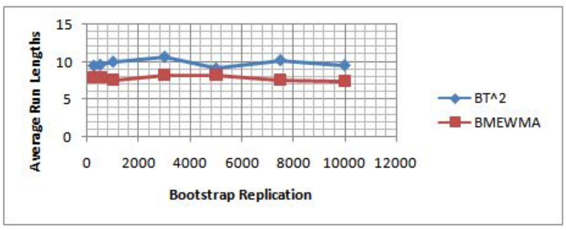

Figure 8: ARL for the Bootstrap Methods Compared

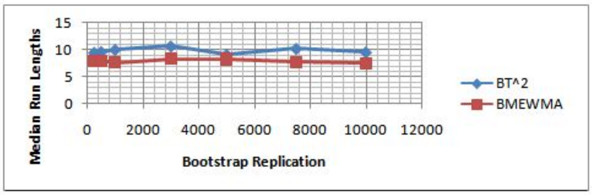

Figure 9: MRL for the Bootstrap Methods Compared

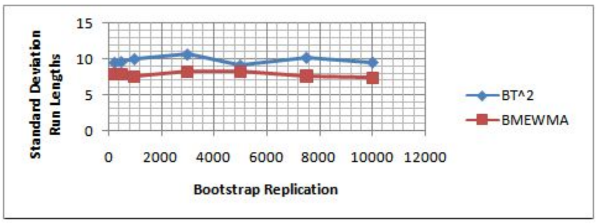

Figure 10: SDRL for the Bootstrap Methods Compared

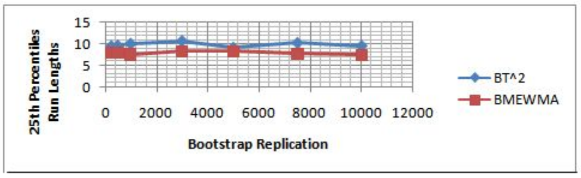

Figure 11: 25th PRL for the Bootstrap Methods Compared

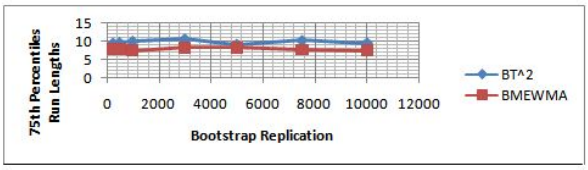

Figure 12: 75th PRL for the Bootstrap Methods Compared

4. Discussion and Interpretation of Results

Meanwhile, bootstrap samples are replicated 250, 500, 1000, 3000, 5000, and 7500, 10000 times from the original data set Oyeyemi (2011). Tables 1 summarizes the control limits for f-distribution T^2, bootstrap T^2 and BMEWMA at chosen values of α = 0.01, 0.025, 0.05, 0.1, 0.2, 0.25. However, Table 2 summarizes the average run length (ARL), median run length (MRL), standard deviation run length (SDRL) and percentiles run length (PRL) from the bootstrap methods. The smaller the value the faster and good such method is in detecting signals. From Tables

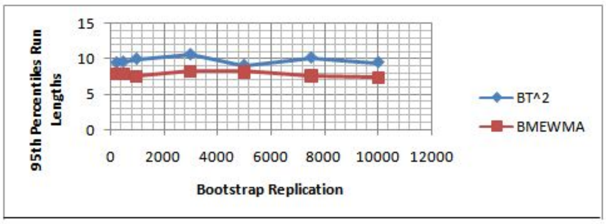

Figure 13: 95th PRL for the Bootstrap Methods Compared

1, it is clear that the proposed bootstrap T^2 and BMEWMA control methods are capable in detecting both large and little signals in any given process effectively. Figures 1-7 shows the results of control limits obtained when the three methods (F-Distribution T^2, Bootstrap T^2 and BMEWMA) are compared. A critical look at the figures shows that BMEWMA has the ability to detect small shift, followed by Bootstrap T^2 and F-Distribution T^2 with the ability to detect large shift. Figures 8-13 shows the results obtained when both Bootstrap T^2 and BMEWMA methods are compared using ARL, SDRL, MRL and PRL. A critical look at the figures shows that BMEWMA has the ability to detect shift first, followed by Bootstrap T^2 ability to detect shift.

5. Conclusion

This study critically looked at the bootstrap Hotelling's T^2 and BMWEMA methods of setting control limits whether or not the underlying distribution is known, and the assumption of multivariate normality is satisfied. Using an empirical data set, the bootstrap results obtained in this study at different alpha (α) levels has been shown to be better than the existing method when compared. Generally, control limits decrease with the increase of alpha (α) levels as shown in Tables 1 and Figures 1-7. This is because it is logically easier to detect a larger shift than a smaller shift. Finally, Table 2 and Figures 8-13 shows the results obtained when both Bootstrap T^2 and BMEWMA methods are compared using ARL, SDRL, MRL and PRL. Therefore, the performance of bootstrap control chart obtained in this study will assist quality managers to take decision for monitoring future production purposes especially in detecting both small and large shift in a process.

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