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Change-point detection : application of Cusum method to real life data

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Abstract. Several procedures have been developed for the detection of abrupt changes in time series. Among these procedures, it can be mentioned the Cumulative Sum (Cusum) type method. It is in such a perspective that Katchekpele et al. (2017) proposed a method using a Cusum type test to detect a change-point in the unconditional variance of the generalised autoregressive conditional heteroskedasticity (GARCH) models. The aim of this paper is to present an application of their technique. After briefly recalling how the test statistic was constructed, the change-point detection algorithm is given and it is shown how it is applied to some real life data.

Key words: change-point; squared Cusum test; GARCH model; application
AMS 2010 Mathematics Subject Classification Objects : 37M10; 62P20

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Résumé. (Abstract in French) Plusieurs procédures ont été développées pour la détection de ruptures dans les séries chronologiques. Parmi ces procédures, on peut citer la méthode des sommes cumulées (Cusum en anglais). C'est dans cette optique que Katchekpele et al. (2017) ont proposé une méthode utilisant un test de type Cusum pour détecter une rupture dans la variance inconditionnelle des modèles GARCH. Le but de cet article est de présenter une application de leur technique. Après avoir brièvement rappelé comment la statistique de test a été construite, l'algorithme de détection de rupture est donné et il est montré comment il est appliqué à certaines données réelles.

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1. Introduction

Change-point analysis plays an important role in the study of time series. Since Page (1955), detection of change-point has received much attention in the literature and it is found in several fields of application (medical science, climate, hydrology, finance, $\cdots$). A large number of articles have been published in various journals. Inclán and Tiao (1994) studied the detection of multiple changes of variance in a sequence of independent observation and give an intuitive basis for the iterated cumulative sums of squares algorithm. Chen and Gupta (2011) gave a considerable amount of new information on recent change-point research results. Irungu et al. (2018) have proposed a change-point estimator based on Mahanttan distance. Katchekpele et al. (2020) developped a Cramér-von Mises (C.V.M.)-type test for a change-point detection in residuals of an ARCH model. For more details on the change-point detection one can for example see Aminikhanghahi et al. (2017), Barassi et al. (2020), Junmo and Jiwon (2020) and the references therein. Several procedures have been developed for the detection of abrupt change. Shi (2020) has proposed a survey of change-point techniques for time series data among which the Cusum procedure introduced by Brown et al. (1975). Following him, numerous works devoted to this procedure, among which Katchekpele et al. (2017) who studied a cumulative sum-type test to detect a change in the unconditional variance of GARCH models. In this work, we are going to explain their version and apply it to real data.

The remainder of the paper is organized as follows. In Section 2, we briefly describe the Cusum method used. Results on application to Ivory Coast BRVM 10 and South Africa Top 40 data set are given in Section 3. We give some concluding remarks in Section 4.

2. Methodology

In this section we present how the Cusum test statistic is constructed and used.

2.1. Cusum test statistic

Katchekpele et al. (2017) observe a stochastic phenomenon $(z_1, z_2, \cdots, z_n)$ which is known to be generated by the square of a GARCH process and study a Cusum-type test to detect a change in the unconditional variance of GARCH models. Their proposed test statistic is

$$Stat = \frac{1}{\tau} \sup_{0 \leqslant t \leqslant 1} |X_n(t)|, \tag{1}$$

where

$$X_n(t) = \frac{1}{\sqrt{n}} \sum_{i=1}^{[nt]} (z_i - \overline{z}). \tag{2}$$

They showed that, under the null hypothesis (no change), the Cusum test statistic converges to the supremum of a standard Brownian bridge and in order to apply their asymptotic result they estimated τ^2, as in Berkes et al. (2009), by

$$\hat{\tau}_n^2 = \frac{1}{n} \sum_{i=1}^{n} (z_i - \overline{z})^2 + 2 \sum_{j=1}^{q} \left(1 - \frac{j}{q+1}\right) \frac{1}{n-j} \sum_{i=1}^{n-j} (z_i - \overline{z})(z_{i+j} - \overline{z}), \tag{3}$$

where q denotes the largest integer smaller than or equal to $(\ln(n))^2$ as in Lee et al. (2003). As already mentioned, in this work, we are going to present an application of the version presented by Katchekpele et al. (2017). The algorithm is given below:

2.2. Cusum algorithm

1. Provide a Time Series $u_1, u_2, \ldots, u_n$, where n is the total number of values.
2. Calculates each $z_i = \left(\frac{u_i - u_{i-1}}{u_{i-1}}\right)^2$ for $i = 1, 2, 3, \cdots, n$,
3. Calculate the average of the values in 2., call that $\overline{z}$.
4. Compute the cumulative sum using the following formula:

$$Cusum(t) = \sum_{i=1}^{[nt]} (z_i - \overline{z}).$$

5. Compute the test statistic using the following formula:

$$Stat = \frac{1}{\tau} max\left(abs\left(\frac{1}{\sqrt{n}} Cusum(t)\right)\right).$$

6. Compare $Stat$ to the critical value and decide.

3. Numerical applications

3.1. Data description

Here we consider data from Ivory Coast BRVM 10 and South Africa Top 40 from March 2006 to March 2021, obtained on the investing.com website. Founded in 2007, investing.com is a platform of tools and information related to the financial markets (real-time quotes, statistics, latest financial news, technical analysis, updated lists of stockbrokers and brokers and many more, economic calendar, tools and calculators). The website also provides breaking news and analysis on 250 stock markets around the world in 44 languages. With over 46 million monthly users, and over 400 million sessions, investing.com is one of the top three financial websites in the world according to SimilarWeb and Alexa. In addition to the global stock markets, investing.com covers commodities, cryptocurrencies, indices, bonds, funds, ETF's and world currencies (see Investing).

The BRVM 10 is one of the two main indices of BRVM, a Regional Stock Exchange serving eight west african countries. It puts together the 10 most active stocks on the market.

South Africa Top 40 (JTOPI) is an index made up with the most active 40 stocks in Johannesburg Stock Exchange. Currently, JTOPI has 35 constituents. For both Ivory Coast BRVM 10 and South Africa Top 40, the index closing price (monthly) is used in this study.

3.2. Applications

In this subsection, we present some results of applying our test procedure on real data examples. Our objective is to see if a change has occurred in the unconditional variance of the returns of these data.

Recall that in practice, a large value of $Stat$ implies a change in the variance. At the level of $\alpha = 0.05$, we reject the null hypothesis, under which no variance change is assumed to occur, if $Stat > 1.358$ (see Inclán and Tiao (1994)).

The results of the tests on each of the data types used are summarised in Fig. 1 (Ivory Coast BRVM 10) and Fig. 2 (South Africa Top 40). For Ivory Coast BRVM 10 data, the value of the test statistics is 1.448757 and for South Africa Top 40 data, the value of the test statistics is 1.383231. Each of the two test statistic values obtained are greater than the critical value. This indicates a rejection of the null

hypothesis.

In summary, our procedure detects a change at the vertical line on both figures below. It corresponds to a volatility change held in July 2009 in Fig. 1 and September 2010 in Fig. 2. Remark that change volatility coincides with the repercussions on African countries of the world economic crisis in 2008-2009. Indeed, the global economic crisis of 2008, sometimes called the Great Recession, was not without impact on Africa. While the initial effects of this financial crisis were slow to be felt in Africa, the impact became clear around the second half of 2009 and continued into 2010. It took with it businesses, mines, jobs, income and livelihoods. The decline in trade flows, the decrease in exports and the increase in import prices, the deterioration of the trade balance are among other consequences of this crisis in Africa. It has also seen a reduction in money transfer by migrant workers, due to the weakening economies of Western countries.

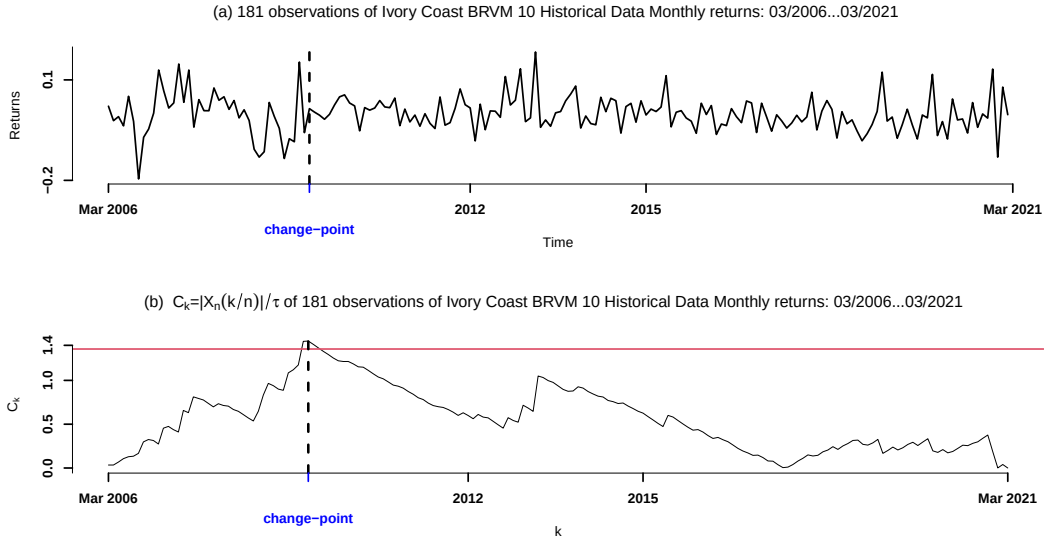

Fig. 1. **(a)** is a Ivory Coast BRVM 10 return 03/2006-03/2021. **(b)** is the corresponding curve $C_k = \frac{1}{\hat{\tau}_n} |X_n(k/n)|$. The horizontal red line represents the limit of the critical region of the test. The vertical dashed line represents the time where the change occurs.

4. Conclusion

In this paper, we have illustrated the application of a change-point detection in volatile series of GARCH models using Cusum-type test proposed by

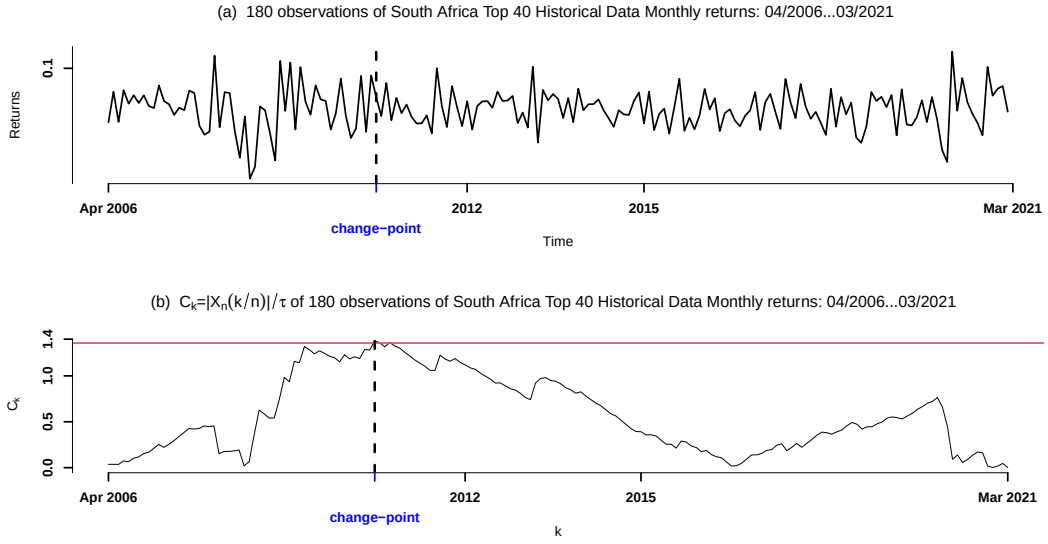

Fig. 2. (a) is a South Africa Top 40 return 04/2006-03/2021. **(b)** is the corresponding curve $C_k = \frac{1}{\hat{\tau}_n} |X_n(k/n)|$. The horizontal red line represents the limit of the critical region of the test. The vertical dashed line represents the time where the change occurs.

Katchekpele et al. (2017). This method, applied to Ivory Coast BRVM 10 and South Africa Top 40 data returns (03/2006 to 03/2021), permits to detect a change in the data ; which can be interpreted as the repercussion of the financial crisis in the world in 2008-2009. The future direction we are going to propose is a procedure to detect multiple changes in GARCH models and a general procedure for change-point detection in multivariate time series.

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