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Modeling the Rwanda Exchange Rates by GARCH Models

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Abstract. . Volatility modeling and forecasts are essential tools to all financial sectors. This paper focuses on weekly exchange rate returns of the *FRW* versus *USD* from 2012 until 2018 obtained from the National Bank of Rwanda. The aim of this paper is to formulate an appropriate *GARCH* model which fits the data. The *GARCH(1,1)* model has been selected after using required techniques of model selection. Parameters have been estimated using Least Squares method first and then validated using *MCMC* method. Once the chain of parameters are found, both visual inspection and basic statistics are computed and in this study, they have illustrated a good compatibility between simulation and observations. Diagnostic of convergence of the chains of parameters has been checked and ensured the model to be accurate. The results obtained from the *LSQ* and *MCMC* methods have been compared and found to be almost similar. An agreement between the model solution and actual data is obtained and a forecast is done by concluding that the estimated values are almost similar to the real data. Hence, the identified model is accepted for forecasting and recommended for further applications.

Key words: Exchange Rate; Volatility; Markov Chain Monte Carlo; Convergence; forecasting
AMS 2010 Mathematics Subject Classification : 00A69; 00A71; 60G10.

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Résumé (Abstract in French) La modélisation et les prévisions de volatilité sont des outils essentiels pour tous les secteurs financiers. Cet article se concentre sur les rendements hebdomadaires des taux de change de la monnaie rwandaise par rapport au dollar américain entre 2012 et 2018, obtenus auprès de la Banque nationale du Rwanda. L'objectif de cet article est de formuler un modèle GARCH approprié qui correspond aux données. Le modèle GARCH(1,1) a été sélectionné après avoir respecté les méthodes idoines en la matière. Les paramètres ont d'abord été estimés à l'aide de la méthode des moindres carrés, puis validés à l'aide de la méthode MCMC. Une fois la chaîne de paramètres trouvée, une combinaison de méthodes statistiques graphiques et quantitatives, illustrent une bonne compatibilité entre la simulation et les observations. Le diagnostic de convergence des chaînes de paramètres a été vérifié et a permis d'assurer la précision du modèle. Les résultats obtenus à partir des méthodes LSQ et MCMC ont été comparés et se sont avérés presque similaires. Le modèle a été retenu en conséquence.

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1. Introduction

Modeling exchange rate has always been a drive truck to success of both investors and policy makers (Oyinlola, 2018). The concern in exchange rate is not random , exchange rate which is the value conversion of one currency in terms of another currency, plays a great role on foreign trade and investments. Added, foreign exchange market covers a large scale of the financial market (Dieu et al., 2015).

Exchange rate perturbations have various impacts on the nation's economy, international trade, economic growth, capital flows, interest rates and even inflation (Ekong and Onye, 2017). The fact that exchange rate volatility is an important tool to measure uncertainty of an economic atmosphere of a country supports the attention of policy makers in accurate forecasts about future values (Epaphra, 2016).

Perturbations in forex boosted financial crisis for different periods of time. The list of the financial decline due to unstable forex is long. Asian countries endured a worse financial status linked to the Russian debt default in 1998, the crisis followed by the fall of the world trade center in 2001. In 2007 – 09 a worldwide financial crisis from exchange rate movements were notified. During 2008 a big number of currencies depreciated while in 2009 the crisis related movements reversed (Murenzi et al., 2015).

Previous researches in terms of exchange rate often revealed heteroscedastic behavior or volatility clustering *i.e.* that after a long run higher volatility tends to cluster together and low volatility tend to cluster together (Héricourt and Poncet, 2015). Therefore, after the Bretton Woods convention, volatility of exchange rates became a source of interest for both policymakers and academics (Héricourt and Poncet, 2015).

The government of Rwanda has embarked on economic reforms since 1995. The foundation of the transformation of the economy is based on a process of economic liberalization and turning away from control regulation and state command to market policies. One of the key areas of the reforms were the exchange rate and external sector competitiveness in Rwanda by introducing a more flexible exchange rate regime and fully liberalized current account (Oyinlola, 2018).

One of the targets among others in time series analysis is to predict future values. The knowledge of exchange rate volatility estimation and forecasting is an important tool for planning as well as system control in many areas such as financial analysis, asset pricing, risk management, quality control, investment analysis, and so forth (Epaphra, 2016).

When a model is estimated, the next step is to identify parameters in the model. They exist several ways to estimate parameters but in this work, Least Square (*LSQ*) and *MCMC* methods have been chosen. *MCMC* method samples from a probability distribution based on constructing a Markov chain that has the desired distribution as its equilibrium distribution. Through a *MCMC* approach a wide range of posterior distributions are simulated and their parameters are found numerically.

The *MCMC* method is governed by the Bayesian inference where all model parameters are random quantities and thus can incorporate prior knowledge. In contrast, for the traditional approach also called frequentist, the statistical inference is done by considering all parameters to be unknown but fixed quantities (Henneke et al., 2007).

Let $Y_1, \ldots, Y_n$ be the observations from a probability model $p(Y_1, \ldots, Y_n|\theta)$ with θ an unknown parameter vector. Therefore, the posterior distribution is computed using Bayes' formula as shown in Equation (1) (Atoi, 2014).

$$p(\theta|Y) = \frac{p(Y_1, \ldots, Y_n|\theta)\pi(\theta)}{\int p(Y_1, \ldots, Y_n|\theta)\pi(\theta)d\theta} \tag{1}$$

In an easy language, Equation (1) is written as

$$P(\theta|Y) = \frac{P(Y|\theta)P(\theta)}{P(Y)}, \tag{2}$$

where, $P(\theta|Y)$ is the posterior distribution of θ, $P(Y|\theta)$ is the likelihood function, $P(\theta)$ is the chosen prior distribution of θ, $\pi(\theta)$ a non informative density and $P(Y)$ is the normalizing constant (Atoi, 2014). Equation (2) means that the posterior distribution is proportional to the product of the likelihood distribution and the prior distribution.

Given a set of set of random draws θ^1, θ^2, θ^3, ..., θ^K, from the posterior distribution. It is always possible to estimate all statistics of interests from posteriors through simulations. For instance mean, mode, median and standard deviation of a certain quantity of interest $m(\theta)$ are obtained by using the sample statistics of values of m(θ^1, θ^2, θ^3, ..., θ^K). The most basic types of *MCMC* algorithms are Metropolis-Hastings algorithm and the Gibbs sampler which will be used in parameters estimation (Rodríguez-Niño, 2000).

Gibbs sampling is a way to turn a high-dimensional problem into several one-dimensional problems and it is the simplest of *MCMC* algorithms that should be used if sampling from the conditional posterior is possible (Henneke et al., 2007; Frühwirth-Schnatter and Sögner, 2009).

The Metropolis algorithm was invented in the $1950s$ by Nicholas Metropolis, from whom the algorithm is named. This algorithm, is a Markov chain Monte Carlo (*MCMC*) method used to obtain a sequence of random samples from a probability distribution (Diaconis and Saloff-Coste, 1998). The Markov chains obtained by using Metropolis-Hastings algorithm has a good mixing of samples because of their reversibility characteristics (Ndanguza et al., 2015).

The Metropolis-Hasting algorithm is a specific *MCMC* that works as follows:

Consider Equation (2) and let $q(x|y)$ be an arbitrary friendly distribution it means we know how to sample efficiently from $q(x|y)$. The conditional density $q(y|x)$ is called the proposal distribution. Thus, the Metropolis-Hasting algorithm as in (Ndanguza et al., 2015). Choose Y_0 arbitrarily.

Given $Y_0, Y_1, \ldots, Y_i$, generate Y_{i+1} as follows:

- Generate a proposal or candidate value $X \approx q(x|Y_i)$.
- Evaluate $r \equiv r(Y_i, X)$ where:

$$r(y,x) = \min\left\{\frac{f(x)}{f(y)}\frac{q(y|x)}{q(x|y)}, 1\right\}$$

- Set $Y_{i+1} = X$ with probability r and $Y_{i+1} = Y_i$ with probability $1 - r$

A simple way to execute step (3) is to generate $U \sim$ Uniform(0, 1). If $U < r$ set $Y_{i+1} = X$; otherwise set $Y_{i+1} = Y_i$. In this case, the proposal density q is symmetric, $q(x|y) = q(y|x)$, and r simplifies to

$$r = \min\left\{\frac{f(X)}{f(Y_i)}, 1\right\}.$$

From construction $Y_0, Y_1, \ldots$ is a Markov chain.

MCMC methods have been successful because they allow one to draw simulations from a wide range of distributions, including many that arise in statistical work, for which simulation methods were previously much more difficult to implement (Kass et al., 1998).

Theoretically, the Markov chains are expected to eventually converge to the stationary distribution, which is also our target distribution. However, we are not certain that our chain will converge after a specific number of draws. It is in this context, that convergence and mixing of parameters diagnostics are applied. Many tests have proven their abilities to assess convergence and mixing characteristics of parameters. In this work we have preferred to use some of them, such as: trace plot, kernel density plot, autocorrelation function plot among others (Ndanguza et al., 2015).

Consider a special case of a bivariate problem (Y, X) with density $f_{Y,X}(y, x)$, then suppose that it is possible to simulate from the conditional distributions $f_{Y|X}(y|x)$ and $f_{X|Y}(x|y)$. Let (Y_0, X_0) be starting values, and assume we have drawn $(Y_0, X_0), \ldots, (Y_n, X_n)$. Then the Gibbs sampling algorithm for getting (Y_{n+1}, X_{n+1}) is:

$$r((Y_n, X_n), (Y_n, X)) = \min\left\{1, \frac{f(Y_n, X) f_{X|y}(X_n|Y_n)}{f(Y_n, X_n) f_{X|y}(X|Y_n)}\right\},$$

where, the current state is (Y_n, X_n) and the proposal is (Y_n, X) with probability $f_{X|Y}(X|Y_n)$. Gibbs sampling iterate until convergence

$$Y_{n+1} \sim f_{Y|X}(y|X_n), X_{n+1} \sim f_{X|Y}(X|Y_n) = \left\{1, \frac{f(Y_n, X)}{f(Y_n, X_n)} \frac{(f(Y_n, X_n)}{f(Y_n, X)}\right\} = 1.$$

This generalizes the obvious way to higher dimensions, where we cycle through the variables, sampling one of them at a time is conditioned on the others.

The aim of this paper is to formulate an appropriate *GARCH* model that fits the Rwanda exchange rate time series data. This study will as well examine the model accuracy for future prediction trust. Most of existing models so far used the *LSQ* and the *MLE* methods to estimate parameters. What if one uses *MCMC* method by taking parameters randomly which vary with a certain prior knowledge? An answer to this question is our main goal whereby at the end, the two methods of parameters estimation are compared to investigate the scale of resulting difference.

The expectations from this paper are to provide a package of skills on both researchers and decision makers about exchange rate volatility in countries struggling to get rid of poverty, Rwanda included. It will help financial policy makers to base their long term projects on built models that ensure a good forecasting performance. As results, formulated macroeconomic policies will be set in accordance with existing models.

The rest of the paper is organized as follows: Section 2, is the method used to identify *GARCH* model fitting the data, followed by Section 3, which is the presentation of results and discussion. The last section is the conclusion followed by references.

2. Methods

This section consists of an introduction to the *GARCH* model and a description of its applications to the real data. We shall take advantages of some statistics and graphical tests to monitor its forecasting adequacy.

Both *ARCH* and *GARCH* model are reasonably good models for analyzing volatility financial time series variables. They are essential tools to capture heteroscedastic behavior or volatility clustering without the requirement of higher order models in various financial markets (Hasanah et al., 2019).

In 1986, Bolleslev proposed *GARCH* model as a generalization of *ARCH* that uses values of the past squared observations and past squared variances to model the variance at time t. A general expression for *GARCH* model is given by (3)

$$r_t = \sigma_t e_t, \tag{3}$$

with σ_t in *GARCH(p,q)* model computed as in (4)

$$\sigma_t^2 = \theta_0 + \sum_{i=1}^{q} \theta_i r_{t-i}^2 + \sum_{i=1}^{p} \beta_i \sigma_{t-i}^2, \tag{4}$$

where, σ_t^2 is the volatility at time t, σ_{t-i}^2 the volatility at time $t-i$, θ_i and β_i are parameters to be estimated and e_t an innovation or shock.

In time series analysis the basic assumption is that data has to be stationary it means that both the mean and variance are constant over time. Instead, if this is not the case we are advised to transform these data (Adhikari and Agrawal, 2013; Ahmed and Cook, 1979). Among other techniques a sequential plot of original data has enough information to assess stationarity. This is done by checking whether the series presents a trend or seasonality. To make data stationary we either use differencing or transformation. Simply, differencing is obtained from subtracting the current data from the previous one as in Equation (5).

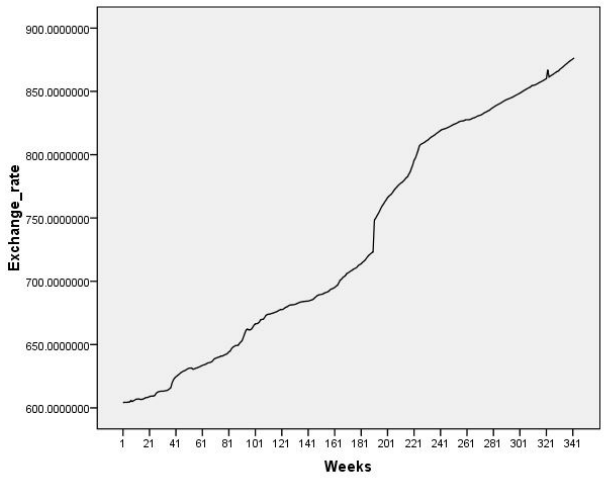

Fig. 1. The exchange rate is plotted against time in the original data time series plot

2.1. Description of data

Used data are secondary from the website of the National Bank of Rwanda (www.bnr.gov.rw). Analysis of data consists of daily average exchange rate of *FRW* versus *USD* ranging from 2012/01/02 to 2018/11/30, roughly seven years. The *USD* is chosen as a bench mark compared to the *FRW* because it is the most converted currency.

The daily exchange rate corresponds to the weight average of trading, selling and buying *USD* ($) in the open market. To reduce the size of data they are converted into weekly data excluding weekends and holidays. Converted data resulted into 342 observations. Software such as *MATLAB*, R programming, *SPSS* and Excel were used to analyze data.

A sequential plot of original time series data is illustrated to investigate whether there is a significant trend or seasonality in series. If ever a trend is noticed, we proceed to differencing data to move towards a stationary data set.

A sequential plot of original time series data is represented by Figure 1.

From observations on Figure 1, unchanged exchange rate time series data illustrates that data are not stationary since there is an upward trend and changing variance with time. However, they were no seasonality because data did not display constant behaviors in particular intervals of time. It is in that context that we accept the null hypothesis H_0 stating that the data are not stationary otherwise the alternative is rejected.

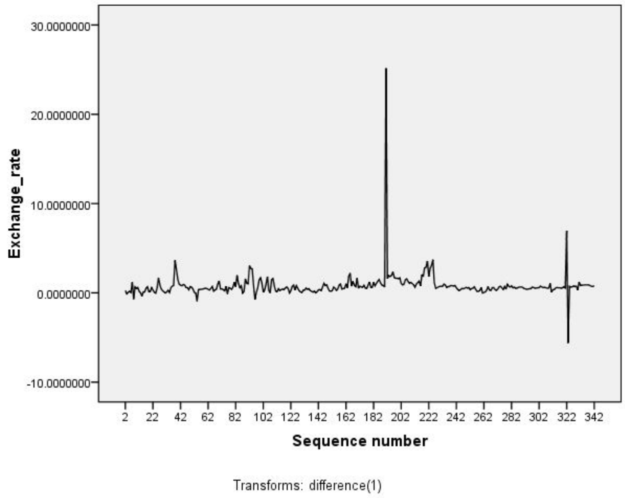

Fig. 2. Sequential plot of transformed time series data

2.2. Transformation of data

Difference is the path towards stationary data. Using Equation (5), we first apply a single difference. In this case we consider $d = 1$, if ever a stationary data is not obtained we proceed with the second difference.

$$W_t = \nabla^d r_t$$

Which is equivalent to:

$$W_t = r_t - r_{t-1} \tag{5}$$

Where r_t is the current return of the exchange rate at time t while r_{t-1} is the return of the previous week. The run sequence time series plot of single difference process displays Figure 2

Figure 2 exhibits a volatility clustering character which means that after a long run higher volatility tends to cluster together and low volatility tend to cluster together. Therefore, we claim heteroscedastic property. Based on the fact that the plot exhibits a horizontal pattern we are convinced that the data is turned to be stationary that is the variance is constant over a zero mean.

2.3. Model identification

In time series analysis the best choice of a model may be easy or difficult. The confusion while choosing a model depends on the fact that several models may

Table 1. Exploratory data analysis of the transformed data. **Abbreviations. min: minimum, max: maximum, std.d: standard deviation, var: variance, skew: skewness, kurt: kurtosis**

Statistics	range	min	Max	Std.d	var	skew	kurt
Values	30.785	-0.6197	25.1660	1.5434	2.382	11.722	184.834

Table 2. *ARMA(p,q)* models main characteristics

	AR(p)	*MA(q)*	*ARIMA(p,d,q)*
ACF	Tails off after lag p	cuts off after lag q	Tails off after lag $q-p$
PACF	Cuts off after lag p	Tails off after lag q	Tails off after lag $p-q$

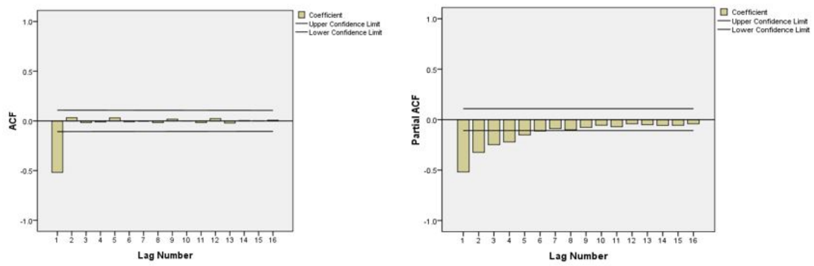

Fig. 3. ACF and PACF plots of first differencing

adequately represent a same time series data set. Table 1 summaries the basic statistics of the first difference of the data.

From Table 1 above, it is noticed that the variance is high 72.726 and standard deviation referred to volatility. Therefore, the higher the standard deviation, the higher the volatility of the exchange market. This high variance warns the existence of heteroscedastic property that the *ACF* and *PACF* plots alone were not able to exhibit. Again, big difference between the maximum and minimum values indicates variability of trend of the return series within the covered period.

Table 2 summaries the main characteristics of *ARMA(p,q)* models that can also be implemented to *GARCH(p,q)* models under some specification.

From Table 2, the orders p and q for *GARCH(p,q)* are identified in a similar way used to determine the orders of an $ARMA$ model of first difference data as depicted by Figure 3.

The *ACF* plot shown in Figure 3 alerts the non presence of correlation in the data from the fact that most of the sample correlations are in the errors range. However, randomly some few sample correlations appear to be outside the range

Table 3. Estimates got by using Least Square method in Parameter estimation

Parameters	θ_0	θ_1	β_1
LSQ Estimated Values	0.6724	0.0501	0.1129

of errors. Thus, r_t is uncorrelated white noise that cannot be modeled linearly.

According to Table 2 and Figure 3, the *PACF* is geometrically decaying and the *ACF* is significant only at lag 1. From the correlogram in Figure 3, the autocorrelation cuts off at lag 1, which means it is a pure moving average of order one, *MA(1)* which was stationary after a single difference.

From the first sight an *IMA(1,1)* model is fitting the data set. However, it is not certain whether this model is enough to warrant consideration since the reason that variance is not constant was ignored.

Analyzing above Tables and Figures, there is a presence of *ARCH* effects, volatility clustering, serial dependence, heavy tailed distribution and high standard deviation. Combining all above details, there is a guarantee that *GARCH(1,1)* model is a good caption of volatility for the considered series. Therefore, Equations (6) and (7) are considered as a model fit.

$$r_t = \sigma_t e_t \tag{6}$$

with

$$\sigma_t^2 = \theta_0 + \theta_1 r_{t-1}^2 + \beta_1 \sigma_{t-1}^2, \tag{7}$$

where σ_t is the variance at time t, σ_{t-1} is the variance at time $t-1$, r_{t-1} is the exchange return of the previous time $t-1$, e_t is a sequence of *i.i.d* random variables with mean zero and unit variance, and θ_0, θ_1 and β_1 are parameters to be estimated. Fortunately a model is obtained, the next step is to estimate parameters by *LSQ* and *MCMC* methods and compare the results from both methods.

3. Results and discussion

3.1. Estimation of parameters by Least Squares method

The least square method to estimate parameters is used and thereafter validate them with *MCMC* methods as shown by Table 3. From Equation (7) parameters to be estimated are θ_0, θ_1 and $\beta 1$.

Inserting the obtained parameters from Table 3 into Equation (7), the following model represented by (8) and (9) respectively is deduced as

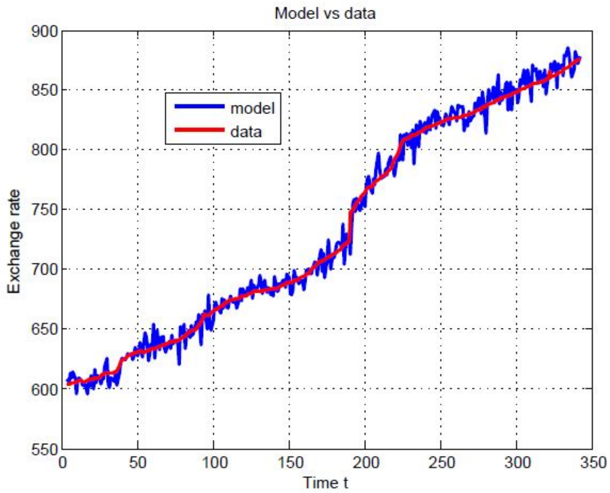

Fig. 4. The fitness of the model to real data of Exchange rate

$$\sigma_t^2 = 0.6724 + 0.0501 r_{t-1}^2 + 0.1129 \sigma_{t-1}^2 \quad (8)$$

and

$$r_t = \sqrt{0.6724 + 0.0501 r_{t-1}^2 + 0.1129 \sigma_{t-1}^2} e_t. \quad (9)$$

3.2. Fitting the estimated model to data

To check how the estimated model fits the real data, a plot that combines the data from the estimated model and the real data are shown by Figure 4.

A visual inspection of Figure 4 reveals that the estimated model marches the real data. Alternatively, Figure 4 is plotted by taking into accounts the *UCL* and *LCL* bounds.

Both Figures 4 and 5 reveal that the real observations are fitted well by the estimated model.

3.3. Markov Chain Monte Carlo method for parameters estimation

MCMC is a higher dimensional computer method of sampling from a distribution (Nilsson, 2016). This step consists in finding the values of the model coefficients which provide the best fit to the data. To carry out a meaningful Bayesian analysis,

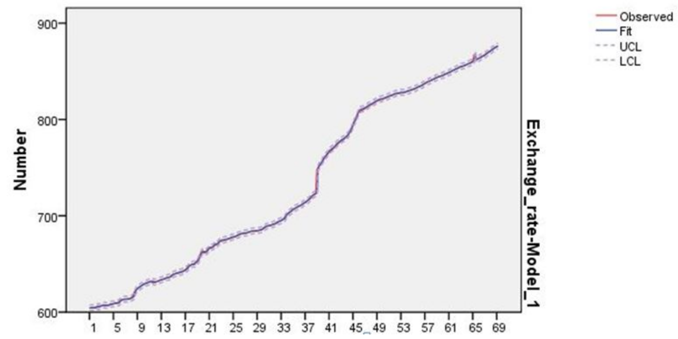

Fig. 5. Real data and simulated ones are plotted for checking the goodness of model fit

Table 4. Posterior mean, mode, median and standard deviation computed by *MCMC* algorithms

Parameters	Mean	Mode	Median	Std	MC-error
θ_0	0.67241	0.6415	0.6724	0.018446	0.0056459
θ_1	0.049811	0.0432	0.0499	0.053633	0.0017803
β_1	0.11296	0.1039	0.1132	0.054027	0.0031658

MCMC were used to estimate parameters from probability distributions.

The convergence of *MCMC* to a stationary distribution always precedes any inference on model parameters. A Visual inspection of below plots will witness both the convergence and mixing properties. Note that the movements of the chain around the parameter space is referred as mixing. To be specific by mixing, the chain samples from the whole stationary distribution instead of a narrow subset of it.

Table 4 provides a hint of agreement between the values obtained from the least square estimation and posterior values of the listed statistics. The fact that these values estimated in both ways are approximately equal in size enhances the goodness of the model fit. However, there is need to check the model validity by diagnostic test.

3.4. Model diagnostic

It involves testing the assumptions of the model to identify any areas where the model is inadequate. For time series data, the selection of the best model fit to the data is directly related to whether residual analysis is performed well. In *MCMC* methods, the key point to diagnose the model is to find a chain that converges to the stationary distribution.

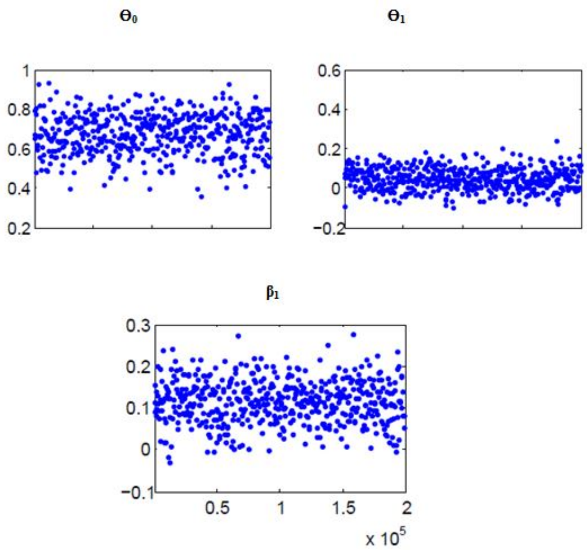

Fig. 6. Chains of parameters time series are plotted in Trace plot

– ***Trace plot***

A crucial tool to explore parameters mixing and the convergence of Markov chain is a trace plot. A trace plot is a graphical representation of the values from the Markov chain against the number of iterations.

From Figure 6, it is noticed that the chain has converged and parameters were mixed well after the burn-in period. This situation was indicated by the free movement of samples during the run time from one region to another in one step. In addition to that, there is a little effect between successive iteration. Therefore, the chain seems to mix quickly.

– ***Pairs plot***

The above Figure 7 displays the pairwise relationship between parameters. The pair plots illustrates that the sampled parameters are notably related. The drawn trace plots of parameters are evidences of converging *MCMC*. A contour plot is a line that joins points of equal elevation. The latter plot results from the projection of a three a dimensional object onto a two dimensional surface.

Basing on Figure 8, there are many points at the center as shown by the bold blue region. The number of these points reduces as we diverge from the center. The central concentration of these points favors the convergence theory. In addition to that, the correlation among pairs of posteriors distribution is weak which means that there is a good mixing.

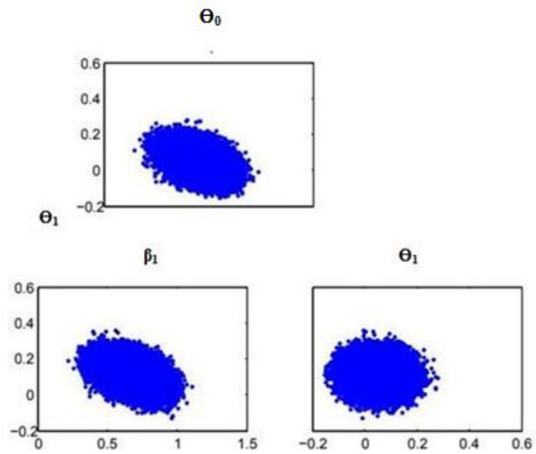

Fig. 7. Parameters are compared to whether there is a relationship in pairs plot

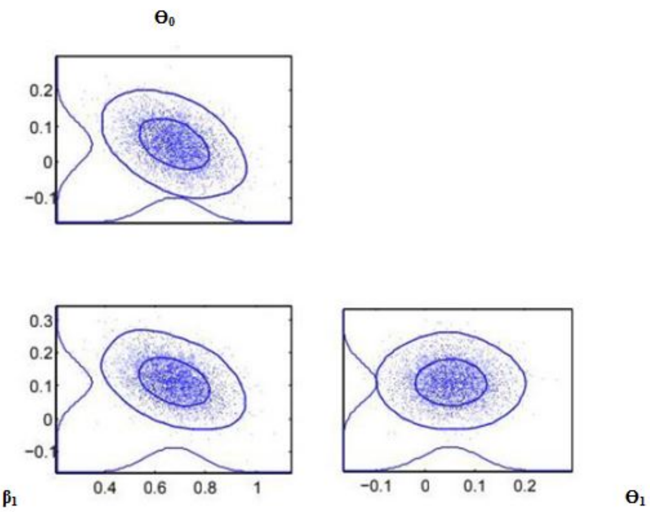

Fig. 8. Confidence contours plot of posteriors distribution

– ***Kernel density plot***

The kernel density curve is a plot that helps to visualize the overall shape of a distribution. The kernel estimation produces a smooth empirical probability density function while an histogram suffers of data binning originating from the lack of continuity Kearney and Daly (1998).

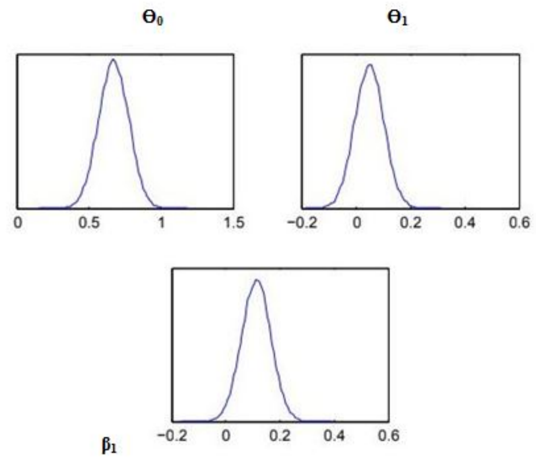

Fig. 9. Kernel density plot for testing the good or bad mixing.

The kernel density plots in Figure 9 represents a bell curve type which shows the normality of parameters. Since kernel density plots alerts a smooth empirical histogram and warns that the distribution of parameters is normal i.e a good mixing of the samples is noticed.

– ***ACF* of coefficients**

The little movement in our draws is justified by the slowly decaying autocorrelation as depicted by the Figure 10. After the exponential decay for certain lags the AC of parameters became stable around zero as the number of lags kept increasing. These behaviors confirm a closer movement of the chain around the parameters space thus a good acceptable mixing that obviously converged after stability.

3.5. Forecasting

One of the targets among others in time series analysis is to predict future values (Uwamariya and Ndanguza, 2018). The knowledge of exchange rate volatility estimation and forecasting is an important tool for planning as well as system control in many areas such as financial analysis, asset pricing, risk management, quality control, investment analysis among others. With a help of *SPSS* we have forecasted the values of exchange rate for one year ahead. Figure 11 depicts the predicted time series exchange rate values for next 54 weeks *i.e.* one year ahead Oyinlola (2018); Adhikari and Agrawal (2013).

A visual observation of the Figure 11 warns that the *USD* exchange rate value will keep increasing compared to the *FRW*. Basing on Table 5, the actual values are

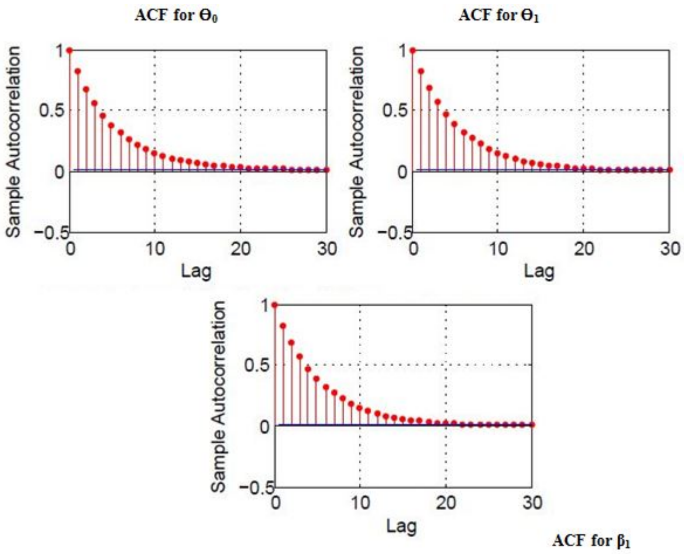

Fig. 10. Autocorrelation plots

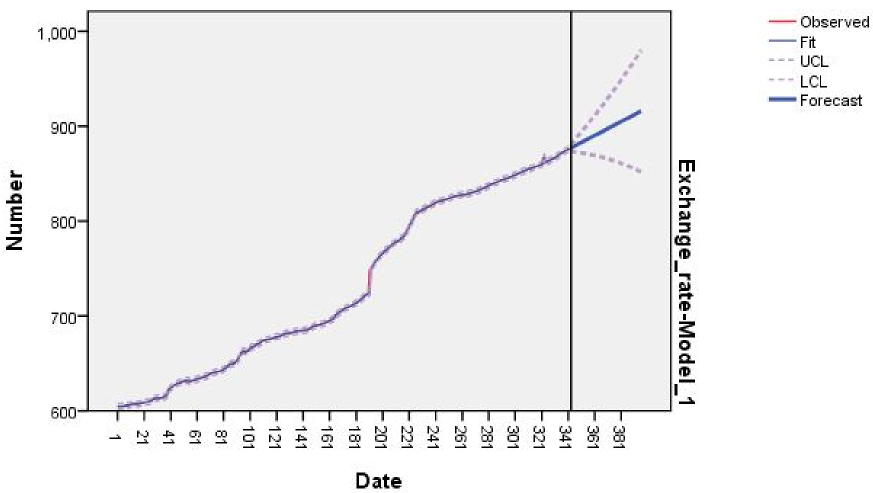

Fig. 11. Model forecast for one year

almost equal to the estimated values. Therefore, the estimated model provides a dominant predictability and fitness of the data which ensured a better forecasting ability.

Table 5. Estimated values against actual values comparison

Time	Actual Value	Estimated Value	Difference
Dec 2018 W1	877.1931774	877.1568325	-0.00363449
Dec 2018 W2	877.8644936	877.8899856	0.0025492
Dec 2018 W3	878.4296316	878.6231388	0.1935072
Dec 2018 W4	878.9399263	879.3562919	0.4163656
Dec 2018 W5	879.100874	880.0894451	0.9885711

Table 5 consists of checking the difference between the actual data and forecasted data. To realize this task, a move beyond 30 November 2018 has been made which was the last observation in this study. Besides, a comparison between the estimated values against the real values for December 2019 is established and fortunately, a slight difference was noticed.

4. Conclusion

The aim of this paper was to find a model that fits the Rwanda exchange rate data from 2012 until 2018 referred to a *USD* as a benchmark. Data were stationary after a single difference and revealed volatility clustering. In addition, a big variance and a large difference between minimum and maximum was obtained which proved the Rwandan currency to be highly volatile compared to the *USD*. Based on the *ACF* and *PACF* of returns, on the presence of *ARCH* effect, volatility clustering, serial dependence, heavy tailed distribution and high standard deviation data were fitted to *GARCH(1,1)* in Equations (8) and (9). Moreover, parameters were estimated using *LSQ* and *MCMC* methods. By the *MCMC* method, thanks to prior knowledge, posteriors have been computed. To check the model accuracy, the *MCMC* trace plot, pairs plot and the *ACF* of the chain parameters were drawn and supported a good mixing and convergence. A comparison between parameters estimated by the *LSQ* and *MCMC* is performed. A good agreement between parameters estimated in both methods is met since they were almost equal in size. The real observations fitted well the estimated model as shown by Figure 4 and Figure 5. The predicted results showed that the estimated exchange rates were almost equal to the actual values as displayed in Table 5. Therefore, the estimated model is good and can be used to forecast the next 54 weeks equivalent to one year ahead. Future works are advised to include weekends, holidays and more than one variable (multivariate *GARCH* models) to ensure the prediction performance is improved.

Conflict of interest

The authors declare that there is no conflict of interest regarding the publication of this paper.

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